SENTINEL-1 DOPPLER AND OCEAN RADIAL VELOCITY (RVL) ALGORITHM DEFINITION

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Resumé / Summary:

This document describes the algorithm implemented in the S1 L2 IPF for estimating the L2 Doppler and the Radial Velocity (RVL) component of the Sentinel-1 Level 2 Ocean (OCN) product.

The L2 Doppler and Radial Velocity component is included in the L2 OCN processing and product together with the Ocean Swell Spectra (OSW) and Ocean Wind Field (OWI) information.

This document is the Sentinel-1 IPF delivery item PAL2-1.

Keywords: Sentinel-1, Doppler Frequency, Radial Velocity, Level 2 Product

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Contents

1 Introduction .................................................. 5
   1.1 Purpose ................................................. 5
   1.2 Scope ................................................ 5
   1.3 Document Structure .................................... 5

2 Documents .................................................... 6
   2.1 Applicable Documents .................................. 6
   2.2 Reference Documents .................................. 7

3 OCN Product Overview ....................................... 8
   3.1 Product Organisation ................................... 9
   3.2 Processing Workflow .................................... 9

4 RVL Component Overview .................................... 12

5 Algorithm input data requirements ......................... 13
   5.1 Antenna Information ................................ 13
   5.2 SLC Data ............................................. 13

6 Doppler and radial velocity estimation algorithm .......... 15
   6.1 Algorithm overview ................................... 15
   6.2 Data .................................................. 15
      6.2.1 Input data ....................................... 15
      6.2.2 Internal data .................................... 18
      6.2.3 Output data ..................................... 18
   6.3 Doppler and radial velocity estimation procedure ....... 18
      6.3.1 Estimation of observed azimuth and range Fourier profiles and initial value of Doppler frequency ........... 19
      6.3.2 Computation of reference profiles .................. 23
      6.3.3 Side-band correction ................................ 23
      6.3.4 Estimation of precision Doppler values: ............ 25
      6.3.5 Resampling to output grid and estimation of radial velocity .......... 26

7 Input Files .................................................. 28
   7.1 SAR Image Products .................................. 28
      7.1.1 SAR Product Annotations ......................... 28
   7.2 Auxiliary Data ......................................... 29
   7.3 Internal Auxiliary Data Files ......................... 29
      7.3.1 Coastline and Land Masking Data - LOP_CLM .... 29
      7.3.2 S-1 Antenna Embedded Row Pattern - LOP_PAT .... 29
      7.3.3 S-1 Antenna Excitation Coefficients - LOP_COE ... 29
      7.3.4 S-1 Antenna LUT - LOP_LUT ....................... 30
      7.3.5 Internal Processing Parameter File - PRM_LOPIn .... 30
   7.4 External Auxiliary Data Files ........................ 30
Acronyms and Abbreviations

ASAR        Advanced Synthetic Aperture Radar
CLM         Coast-Line and Land masking
CLS         Collecte Localisation Satellites
CNR         Clutter to Noise Ratio
dB          Decibel(s)
DC          Doppler Centroid
DCE         Doppler Centroid Estimate
ECMWF       European Centre for Medium-range Weather Forecasts
ENVISAT     ENVironment SATellite
ESA         European Space Agency
EW          Extra Wide
HH          Horizontal polarisation on transmit and receive
IDL         Interactive Data Language
IM          Image Mode (ASAR)
IPF         Instrument Processing Facility
IW          Interferometric Wide
IWS         Interferometric Wide Swath
L1          Level 1
L2          Level 2
MDA         MacDonald, Dettwiler and Associates Ltd.
MET         Meteorological
NESZ        Noise Equivalent Sigma Zero
NRCS        Normalised Radar Cross Section
Norut       Northern Research Institute
NWP         Numerical Weather Prediction
<table>
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<tr>
<th>Abbreviation</th>
<th>Full Form</th>
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<tr>
<td>OSW</td>
<td>Ocean Swell Wave Spectra</td>
</tr>
<tr>
<td>OWI</td>
<td>Ocean Wind Field</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PRF</td>
<td>Pulse Repetition Frequency</td>
</tr>
<tr>
<td>R&amp;D</td>
<td>Research and Development</td>
</tr>
<tr>
<td>RMS</td>
<td>Root Mean Square</td>
</tr>
<tr>
<td>RMS&lt;sub&gt;e&lt;/sub&gt;</td>
<td>Root Mean Square Error</td>
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<tr>
<td>RVL</td>
<td>Radial Velocity Field</td>
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<tr>
<td>S-1</td>
<td>Sentinel-1</td>
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<td>S/C</td>
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<tr>
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<td>Stripmap</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
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<tr>
<td>SOPRANO</td>
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<tr>
<td>SRR</td>
<td>System Requirements Review</td>
</tr>
<tr>
<td>TBC</td>
<td>To Be Confirmed</td>
</tr>
<tr>
<td>TBD</td>
<td>To Be Determined</td>
</tr>
<tr>
<td>TOPS</td>
<td>Terrain Observation with Progressive Scans</td>
</tr>
<tr>
<td>V</td>
<td>Vertical</td>
</tr>
<tr>
<td>VV</td>
<td>Vertical polarisation on transmit, Vertical polarisation on receive</td>
</tr>
<tr>
<td>WS</td>
<td>Wide Swath</td>
</tr>
<tr>
<td>WSM</td>
<td>Wide Swath Mode</td>
</tr>
<tr>
<td>WV</td>
<td>Wave Mode</td>
</tr>
<tr>
<td>WVW</td>
<td>Wave Mode Ocean Spectra Level 2 Product</td>
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</table>
1 Introduction

1.1 Purpose

The objective of this document is to define and describe the algorithm implemented in the S1 L2 IPF and the processing steps for the generation of the Radial Velocity (RVL) component of the Sentinel-1 Level 2 Ocean (OCN) product.

1.2 Scope

The OCN product contains three components: the Ocean Swell spectra (OSW) component, the Ocean Wind Field (OWI) component, and the Radial Surface Velocity (RVL) component. These three components are all merged into a common OCN product for the Wave Vignette (WV) and Strip Map (SM) modes. For the S-1 TOPS mode, the OCN product consists of only the RVL and OWI components. A description on how all these three components (OSW, OWI, RVL) are connected into the L2 ocean processing is outlined in Section 3. This document contains only the RVL algorithm definition. The OWI and OSW algorithm definitions are provided in separate documents [A-8], [A-9].

This document satisfies the PAL2-1 deliverable defined as per the content defined in the Sentinel-1 IPF Statement of Work [A-1],[A-2] for review at the Sentinel-1 IPF Preliminary Design Review (PDR L2) and Critical Design Review (CDR L1 & L2).

1.3 Document Structure

This document is structured as follows:

Section 1 introduces the purpose, scope, structure and conventions of the document

Section 2 lists the applicable and reference documents

Section 3 gives a contextual overview of the L2 OCN processing and component

Section 4 gives a short L2 RVL component overview

Section 5 describes RVL algorithm input data requirements

Section 6 describes the Doppler and radial velocity estimator

Section 7 lists and describes all the input files

Section 8 lists the main symbols used in the RVL algorithm

Section 9 lists the content of the output netCDF product file

Appendix A gives the theoretical background for the algorithm
Appendix B describes the processing requirements for the L1 SLC product.

Appendix C outlines the best (idealized) theoretical performance of the algorithm.

Appendix D outlines background theoretical derivations of idealized 2D-antenna model and antenna errors, and impacts on Doppler estimate.

Appendix E includes figures related to Appendix B.

2 Documents

2.1 Applicable Documents


A-2 Contract Change Notice N.2, Changes in ESRIN Contract No. 21722/08/ILG, June 21, 2010


A-8 S1-TN-CLS-52-9049 Sentinel 1 Ocean Wind Field (OWI) Algorithm Definition, Issue/Revision 1/2, April 27, 2011, CLS


A-10 S1-DD-ASD-PL-0003, Issue 4, March 18 2011, Sentinel-1 SAR Instrument Antenna Model Description

A-11 http://www.unidata.ucar.edu/software/netcdf/
2.2 Reference Documents

The following documents provide useful reference information associated with this document. These documents are to be used for information only. Changes to the date/revision number (if provided) do not make this document out of date.


R-3 BOOST Technologies, SAR WINDS WAVES CURRENTS Validation technical notes, Technical note (WP6), BO-024-ESA-0408-VTN, version 1.0, 09/08/2006

R-4 Chapron B., Collard F., Arduhin F., ”Direct measurements of ocean surface velocity from space: Interpretation and validation“, Journal of Geophysical Res., 110, C07008, 2005

R-5 Mouche A.A., Chapron B., Reul N., Collard F., ”Predicted Doppler shifts induced by ocean surface wave displacements using asymptotic electromagnetic wave scattering theories”, Waves in Random and Complex Media, Volume 18, Issue 1, February 2008, pp. 185


R-10 Antenna Doppler Contribution Predictor Preliminary Validation, Doc. No.: SAR-EQWG-132-TEN, Aresys, Sept. 2010
3 OCN Product Overview

The level-2 (L2) ocean product (OCN) has been designed to deliver geophysical parameters related to the wind, waves and surface velocity to a large panel of end-users. The L2 OCN products are estimated from Sentinel-1 (S-1) Synthetic Aperture Radar (SAR) level-1 (L1) products. L2 OCN products are processed by the Level 2 IPF processor and benefit from robust and validated algorithms [R-3]. A diagram of the L2 Ocean processing unit context is presented in Figure 1. In this figure, external IPF interfaces have a white background, internal IPF interfaces are identified by a grey background, and interfaces with a yellow background are only applicable when the L2 processor is used in test mode outside of the normal IPF environment.

The processor can be used in PDGS environment or in a stand-alone HMI mode. In both cases, a job order is read by the processor to get all high level information required for processing a particular product (e.g. names and directories of input L1 files, names and directories of auxiliary data files, directories of outputs files, etc). Processing then starts from L1 products using the auxiliary data files provided (e.g. the L2 processor parameter file). During the processing, a log file is generated to monitor the status of each processing step. The final step of the processing is the creation of the product including writing of all the geophysical information into netCDF files.

![Figure 1: Sentinel-1 L2 Ocean Processing Context Diagram](image)
3.1 Product Organisation

Each L2 OCN product contains up to three geophysical components: the radial velocity (RVL), the ocean surface wind field (OWI) and the ocean swell wave spectra (OSW) components. These components are formatted into one output netCDF file. For SM and WV modes, the L2 product contains all three components. For TOPS mode, the product contains only RVL and OWI components. The detailed algorithm definition of each component is described in a dedicated document (this document is for the RVL and [A-8], [A-9] are for the OWI and OSW). The outputs variables related to each component are listed and defined in the product definition documentl [A-5].

For the SM and TOPS modes, the information related to each component is estimated onto a specific grid cell (ground range) whose properties are chosen to optimize the inversion schemes. As a consequence, the SM mode output netCDF file has three components and the TOPS mode output netCDF file has two components, each set having its own resolution. In addition, the most pertinent geophysical parameters from RVL and OSW components are interpolated onto the OWI grid to get a set of variables defined at the same resolution. The default value for the resolution of this common grid is 1 km for SM and TOPS modes. The set of variables from RVL and OSW interpolated onto OWI grid is listed in [A-5], [A-8] Section 8. RVL and OSW are estimated from L1 SLC internal product. OWI is estimated from L1 GRD internal product. For WV mode, there is no grid. In this case, the resolution of the components is simply the size of the imagette: 20 km. The three components are estimated from L1 SLC internal products.

3.2 Processing Workflow

For SM and TOPS modes, the components are estimated independently. This means that for a given acquired scene, the steps for each component are:

- the appropriate L1 internal product is read,
- the variables corresponding to the considered component are estimated
- a temporary file containing the results is saved locally.

For each component, these three latter steps are executed by different IDL scripts based on the same library of IDL functions. These 3 scripts are coordinated by a Python script which collects all information mandatory for L1 processing of each component. Then, when it is completed for all components, the components outputs and logging files are merged into a single netCDF file and a single logging file by another script. For WV mode, the three components are estimated sequentially from the same L1 SLC internal product with the same IDL script. The SM and TOPS modes have the dual-polarization option. However, the L2 OCN components are always estimated only using the information from the co-polarized signal. Thus, the algorithms for each component as well as the workflow for the L2 OCN product generation are not different from that of single polarization product. The OCN Product consists of three components (OWI, OSW
and RVL), and these three components are derived through three different processing algorithms:

- The Radial Velocity algorithm, as described by this document
- The Ocean Swell Spectral algorithm, as described [A-9]
- The Ocean Wind Field algorithm, as described in [A-8]

Table 1 presents a summary of which components are included in the OCN Product per acquisition mode, the L2 processing algorithm used to calculate the values for that component, and the type of input L1 product (either SLC or GRD) is required by each algorithm. Further details about these L1 GRD and SLC L1 products can be found in the respective algorithm documents.

<table>
<thead>
<tr>
<th>Acquisition Mode</th>
<th>L2 OCN Product Component</th>
<th>Input L1 Product</th>
<th>L2 Processing Algorithm</th>
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<tr>
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<td>OWI</td>
<td>GRD</td>
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<td></td>
<td>RVL</td>
<td>SLC</td>
<td></td>
</tr>
<tr>
<td>IW</td>
<td>OWI</td>
<td>GRD</td>
<td>✓</td>
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<tr>
<td></td>
<td>RVL</td>
<td>SLC</td>
<td>✓</td>
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<tr>
<td>EW</td>
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<td>SLC</td>
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<tr>
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<td>SLC</td>
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The L2 processing algorithms support the processing of both Sentinel-1 and ASAR L1 products that have been produced by the S1 IPF in the Stand-Alone environment. In the Test Mode of the L2 OCN Processor, they can also use L1 products that have been produced by the PF-ASAR processor, and therefore follow the ENVISAT/ASAR product format. Table 2 shows the acquisition modes, sensors and input product types supported by each of the three L2 processing algorithms.
<table>
<thead>
<tr>
<th>L2 Processing Algorithm</th>
<th>Acquisition Mode</th>
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<th>L1 Processor</th>
<th>L1 Product Type</th>
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<td>PF-ASAR</td>
<td>IMS</td>
</tr>
<tr>
<td></td>
<td>WV</td>
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<tr>
<td>Ocean Wind Field</td>
<td>SM</td>
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<td>S1 IPF</td>
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<td>EW</td>
<td>Sentinel-1</td>
<td>S1 IPF</td>
<td>SLC</td>
</tr>
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</table>
4  RVL Component Overview

The Sentinel-1 SAR can be operated in one of four nominal acquisition modes:

- Stripmap Mode (SM)
- Interferometric Wide-swath Mode (IW)
- Extra-Wide-swath Mode (EW)
- Wave Mode (WV)

The Sentinel-1 ocean Doppler and radial velocity processing supports all the modes listed above. The Sentinel-1 Level 2 RVL component consists of an estimate of the total Doppler frequency [Hz] and the corresponding radial velocity [m/s] estimated from a Sentinel-1 Level 1 Single-Look Complex (SLC) SAR image. The RVL component contains an estimate of the width of the ocean Doppler spectra. The Doppler width is a new geophysical parameter that has never been estimated from SAR data before. For each of these parameters the RVL component contains the corresponding standard deviation of the estimates. The image from which a single RVL is computed can be a SLC imagette from the WV mode, or sub-image extracted from a SM SLC image, IW SLC image, or an EW SLC image. The RVL product is given on a grid similar as to the OSW or OWI components.

The Doppler frequency is estimated from the SLC data by fitting (least square minimization) the antenna model to the observed azimuth spectra taking into account effects from additive noise and side band effects. The Doppler frequency is the total estimated Doppler frequency offset without any geometric or mispointing corrections. The corresponding radial velocity is retrieved from the Doppler frequency after correcting the estimated total Doppler frequency for antenna mispointing as function of elevation angle and compensating for the attitude/orbit Doppler signal error. As for the OSW and OWI component the RVL component contains also information on the block size used for estimation. The spatial coverage of the RVL product is equal to the spatial coverage of the corresponding L1 WV-SLC or sub-images extracted from the L1 SM/IW/EW-SLC products, limited to ocean areas.
5 Algorithm input data requirements

The RVL processing algorithm requires access to S-1 antenna error matrix in order to generate internally the radiation beam patterns [A-10]. The error matrix data are provided as an IPF auxiliary file. The RVL algorithm also set some specific requirements to the internal SLC that will be generated for the L2 processing. These are specified below and the reason for needing them is justified.

5.1 Antenna Information

The RVL algorithm models the contribution from aliasing of energy from neighbouring areas into the estimation area (side band effects). In case of intensity gradients in the image (which is very likely to happen over ocean areas) this will cause slightly asymmetric azimuthal profile. Although small, this asymmetry will cause a significant bias in the Doppler frequency estimate. In order to model this effect the RVL algorithm models the behaviour of the antenna during acquisition including the side bands. And for TOPS mode the knowledge of antenna information for the various steering angles is required.

One option to provide the antenna information to the RVL algorithm is to provide the full 2D antenna pattern. However, for TOPS mode this means that the patterns for all steering angles must be provided, resulting in a large amount of auxiliary input data. The solution is to synthesize the 2D antenna pattern in the L2 RVL processing for various swaths and steering angles (TOPS) including all the side bands following the ideal antenna model approach described in [A-10],[R-10]. This require access to the error matrix, the embedded row pattern, the excitation coefficients, and the corresponding association of the excitation coefficients with the PRI within a burst (LUT). The excitation coefficients, the error matrix and the embedded row patterns can be provided as internal auxiliary data, and also the LUT indexes since they will also be kept mostly constant from product to product of same swath/mode.

The requirements are summarized:

- access to antenna error matrix
- access to the excitation coefficients
- access to embedded row patterns
- access to LUT indexes (for TOPS for each burst)

5.2 SLC Data

The RVL algorithm requires SLC data where the data is focused in range such that the additive noise is kept white. This means no weighting or windowing applied during
data focusing. An internal SLC product, that differs from the external SLC product, is developed to meet these requirements. The reason for this requirement is that the algorithm is estimating the additive noise from the range profile, independently from the azimuth profile. In this way the RVL algorithm derives both the Doppler frequency and the geophysical Doppler width from the azimuth profile fitting procedure. This process makes also the fitting procedure more robust.
6 Doppler and radial velocity estimation algorithm

In this section the Doppler and radial velocity estimation algorithm is described step by step. The steps outlined below are described in detail in sub-section 6.3.

6.1 Algorithm overview

The procedure consists of five main parts:

I: Estimation of Observed azimuth Fourier Profile $\tilde{P}$ initial values of signal energy $a$ and Doppler offset $\varpi_{dc}$. This process is done for each burst for IW/EW and for SM/WV in SLC products, on an equidistant grid in zero Doppler SAR coordinates. From the range direction Fourier profiles (observed and model) the additive noise level estimated outside the bandwidth of the signal (Figure 3). The output of this procedure is stored in the array (EST) of estimation point structures.

II: For each estimation point in one burst, the reference Fourier profile $P$ is computed. The resulting profiles are stored in the array (REF) of reference functions. This array is assumed to be fixed and will be used in the precise parameter estimation from several bursts.

III: The weighted sum of the aliased side-bands is computed using the signal energy of the corresponding neighbor areas, and subtracted from the observed profile $\tilde{P}$ (all based on previous estimated values from the EST array). The side-band corrected profile is saved in the EST array.

IV: From the side-band corrected observed profile (in the EST array) combined with the reference profile (from the REF array), the precise estimates of $a$ and $\varpi_{dc}$ are computed and stored in the structure of the estimation point array (EST).

V: From the values stored in the estimation point array (EST) the output product is generated on a given output grid and the radial velocity is computed, combining all swaths (for TOPS) and stored in the output array (RET).

The procedure is outlined in Figure 2.

6.2 Data

In this section the input data, the main internal data and the final output data are described.

6.2.1 Input data

SLC: Data product where the following additional information is included:
Figure 2: Flowchart of RVL Algorithm
- Number of bursts ($N_{\text{bursts}}$) in input SLC product (TOPS)
- Size ($M_{\text{raw}}, N_{\text{raw}}$) of each raw-data burst
- Start-time ($t'^0, \tau'^0$) of each raw-data burst
- Sampling-rate ($\omega_{\Delta}, \bar{\omega}_{\Delta}$) of each raw-data burst
- Azimuth steering-rate ($\gamma$) of antenna.
- Array[$N_{\text{raw}}$] containing the reference between line-number in raw-data burst and antenna T/R coefficient LUT index.
- Array[* $N_{\text{rep}}$] containing the replica pulses (only a small subset is needed).

The SLC data must be processed with no weighting functions in the Fourier-domain. This also includes the range compression transfer function, whose absolute value has to be constant for the whole Fourier-domain (also outside the signal bandwidth).

The following parameters are extracted from the auxiliary antenna data products:

$N_a$ : number of antenna elements in azimuth.

$N_e$ : number of antenna elements in elevation.

$Z^t$ : Array[$N_e, N_a, *$] containing the antenna transmission (exitation) coefficients.

$Z^r$ : Array[$N_e, N_a, *$] containing the antenna receive (exitation) coefficients.

$\delta Z^t$ : Array [$N_e, N_a$] containing the multiplicative weighting (error) coefficients to the antenna transmission coefficients.

$\delta Z^r$ : Array [$N_e, N_a$] containing the multiplicative weighting (error) coefficients to the antenna receiving coefficients.

$f$ : Array[$N_e$] containing the radiation pattern of the T/R modules (embedded row pattern).

L2 AUX_PP2 : L2 processor parameter auxiliary data:

- $RaEst$ : Size of Doppler estimation block in range [m]
- $AzEst$ : Size of Doppler estimation block in azimuth [m]
- $RaRes$ : Size of output grid cell interval in range direction [m]
- $AzRes$ : Size of output grid cell interval in azimuth direction [m]
6.2.2 Internal data

EST : Array[$M_{est}, N_{est}, N_{bursts}$] containing the following structure:

$M_{est}$ : Number of estimation blocks in range

$N_{est}$ : Number of estimation blocks in azimuth

$a$ : Scalar containing the estimated energy of the signal.

$b$ : Scalar containing the estimated energy of the additive noise in the signal.

$\omega_{dc}$ : Scalar containing the estimated angular Doppler frequency offset.

$\vartheta$ : Scalar containing the estimated width of the Doppler frequency spread.

$\hat{P}$ : Array[$N$] containing the observed azimuth Fourier profile.

$\hat{R}$ : Array[$M$] containing the observed range Fourier profile.

REF : Array[$M_{est}, N_{est}$] containing the following structure:

$P$ : Array[$N$] containing the reference azimuth Fourier profiles.

$R$ : Array[$M$] containing the reference range Fourier profiles.

Here ($M_{est}, N_{est}$) is the number of estimation blocks (range, azimuth) (per burst for TOPS). Note that the number of estimation points is much larger than the number of output grid cells. These numbers will depend on the input parameters (block sizes) and imaging mode and must be computed for each case. $N$ is the number of pixels in the reference and observed azimuth Fourier profile. $M$ is the number of pixels in the reference and observed range Fourier profile.

6.2.3 Output data

RET : Array[*,*] containing the following structure:

$\omega_{dc}$ : Scalar containing the estimated angular Doppler frequency.

$\vartheta$ : Scalar containing the estimated width of the Doppler spread.

$U_r$ : Scalar containing the estimate of radial velocity.

$\sigma^2_{\omega_{dc}}$ : Scalar containing the variance of the estimate of $\omega_{dc}$.

$\sigma^2_{\vartheta}$ : Scalar containing the variance of the estimate of $\vartheta$.

$\sigma^2_{U_r}$ : Scalar containing the variance of the estimate of $U_r$.

6.3 Doppler and radial velocity estimation procedure

In this section the five main steps of the algorithm are described. The background theory for the formulas used in the algorithm description can be found in Appendix A. The formulas of the algorithm and the procedure described in the following sections are applicable to TOPS, SM and WV modes. For SM and WV the only difference is that the angular steering rate, $\gamma$ is set to zero.
6.3.1 Estimation of observed azimuth and range Fourier profiles and initial value of Doppler frequency

This subsection refers to Step I of the procedure outlined in Section 6.1.

Input parameters: Observed complex image product: $I_c(t, \tau)$

Output parameters: Observed azimuth spectral profile: $\tilde{P}(\omega)$, Variance of azimuth spectral profile: $\text{Var}\{\tilde{P}\}$, signal energy: $a$, additive noise: $b$, initial value of Doppler frequency: $\varsigma$, weight factor in range: $\chi_{ra}$, weight factor in azimuth: $\chi_{az}$.

Procedure: The procedure below is done for all estimation blocks within the product. The procedure for block extraction from input SLC product is part of the reader, and similar to the one used for the OSW stripmap processing. The reader provides also all the SLC header informations, as well as all the necessary geometry/satellite information (incidence angle, latitude, longitude, radar velocity, heading, etc) for each of the estimation blocks. These parameters are thus not specifically mentioned in the Input Parameter descriptions for the various procedures. The equations refered in the procedure below (and implemented in the code) are given in the Background Equations section below. Initially all weight functions are set to one ($W_P = 1, W' = 1, W_R = 1$), and then the following steps are done:

- compute the windowing functions in Fourier domain, $\hat{h}_{az}$ and $\hat{h}_{ra}$
- compute the weight factors, $\chi_{ra}, \chi_{az}$, using eq.(21)
- compute model of range fourier profile, $R$, using eq.(7)
- compute estimates of energy, $a$, and noise, $b$ using eqs.(16), (17)
- compute estimate of range, $\tilde{R}(\omega)$, and azimuth, $\tilde{P}(\omega)$ fourier profiles and corresponding variances, $\text{Var}\{R\}, \text{Var}\{\tilde{P}\}$ using eqs.(9), (10), (22), (23)
- compute model of azimuth fourier profile, $P$, using $\tilde{P}(\omega)$
- recompute weight functions $W'$ and $W_P$ using eqs.(19) and (24), and repeat the last four steps twice

The initial value of the frequency shift $\varsigma$ of $\tilde{P}$ is estimated using a standard cyclic method:

$$\hat{\varsigma} = \frac{1}{\Delta \tau} \text{Arg}\left\{\int d\omega \tilde{P}(\omega) e^{i\omega \Delta \tau}\right\}. \quad (1)$$

The initial Doppler offset is related to this shift as described in eq (28) and together with the estimated values of $a$, $b$ and $\tilde{P}$ they all are stored in the structure of the estimation point array EST. The initial values of Doppler offset and signal energy are used in the side-band correction procedure described in sub-section 6.3.3.
**Background Equations:** First the 2D-Fourier-profile is estimated from the complex image $I_c$ multiplied with a windowing function and with the dechirping function (for TOPS) as follows:

$$
\tilde{I}(\omega, \varpi) = \left| \int dt \: d\tau \: e^{-i(\omega t + \varpi \tau)} I_c(t, \tau) e^{i \frac{\beta \gamma}{\omega + \varpi} \tau^2} h(t - \bar{t}, \tau - \bar{\tau}) \right|^2
$$

where $I_c$ is the complex image, $\gamma$ is the angular azimuth steering rate of the antenna (given as input data), $h$ is the dyadic constructed spatial window function

$$
h(t, \tau) = h_{ta}(t) h_{az}(\tau)
$$

(TBD) centered at range and azimuth position $\bar{t} = (\bar{t}, \bar{\tau})$, and $\beta$ is the angular signal Doppler rate (in rad/s$^2$) computed using the relation:

$$
\beta = \frac{\nu^2 \Omega}{t}
$$

where $\Omega(\omega, t) \equiv \sqrt{\omega_0^2 + \varpi^2 - \frac{2 \varpi}{\nu}}$ and $\nu(t) = \frac{2 \varpi}{\nu}$. Here $v_r$ is the effective radar velocity, and $\omega_0$ is the carrier frequency (rad/s). Eq. 3 needs only be computed at the estimation center position, $t = t$. We assume the following model for the mean and variance of the 2D-model:

$$
\begin{align*}
\mathbb{E}\{\tilde{I}(\omega, \varpi)\} &= a R(\omega) P(\varpi) + b, \\
\text{Var}\{\tilde{I}(\omega, \varpi)\} &= (a R(\omega) P(\varpi) + b)^2,
\end{align*}
$$

where the model range and azimuth Fourier profiles satisfies:

$$
\int d\omega \: R(\omega) = \int d\varpi \: P(\varpi) \equiv 1.
$$

Let $\{s^{(1)}_{rep}(t) \ldots s^{(N_{rep})}_{rep}(t)\}$ represent the array of pulse replicas, then the model range Fourier profile is computed by:

$$
R(\omega) = \frac{\langle \hat{h}_{ta} \rangle^2 \otimes R_0(\omega) \rangle(\omega)}{\int d\omega' \langle \hat{h}_{ta} \rangle^2 \otimes R_0(\omega') \rangle(\omega')},
$$

where $\hat{s}^{(n)}_{rep}$ and $\hat{h}_{ta}$ are the Fourier transforms of $s^{(n)}_{rep}$ and $h_{ta}$, respectively, and

$$
R_0(\omega) = \sum_{n=1}^{N_{rep}} \left| \hat{s}^{(n)}_{rep}(\omega) \right|^2.
$$

We define the observed range and azimuth Fourier profiles as

$$
\begin{align*}
\hat{R}(\omega) &= \int d\varpi \: \tilde{I}(\omega, \varpi) W_R(\omega, \varpi), \\
\hat{P}(\varpi) &= \int d\omega \: (\tilde{I}(\omega, \varpi) - \hat{b}) W_P(\omega, \varpi).
\end{align*}
$$
where \( \hat{b} \) is an estimator of \( b \), and \( W_R \) and \( W_P \) are positive weight functions with constraints
\[
\int d\varpi W_R(\omega, \varpi) P(\varpi) = 1, \tag{11}
\]
\[
\int d\omega W_P(\omega, \varpi) R(\varpi) = 1. \tag{12}
\]
The means of the profiles are
\[
E\{ \hat{R}(\omega) \} = a R(\omega) + b', \tag{13}
\]
\[
E\{ \hat{P}(\varpi) \} = a P(\varpi) + (b - E\{ \hat{b} \}) \int d\omega W_P(\omega, \varpi), \tag{14}
\]
where \( b' \) is related to \( b \) by
\[
b' \equiv b \int d\varpi W_R(\omega, \varpi). \tag{15}
\]
we observe that if \( \hat{b} \) is unbiased the last term of of equation (14) is zero. Unbiased estimators for both \( a \) and \( b' \) are
\[
\hat{a} = \int d\omega \hat{R}(\omega) g_a(\omega) W'(\omega), \quad g_a(\omega) \equiv \frac{\gamma_0 R(\omega) - \gamma_1}{\gamma_0 \gamma_2 - \gamma_1^2}, \tag{16}
\]
\[
\hat{b}' = \int d\omega \hat{R}(\omega) g_b(\omega) W'(\omega), \quad g_b(\omega) \equiv \frac{\gamma_2 - \gamma_1 R(\omega)}{\gamma_0 \gamma_2 - \gamma_1^2}, \tag{17}
\]
where
\[
\gamma_{\ell} \equiv \int d\omega R^\ell(\omega) W'(\omega) \tag{18}
\]
for all positive choice of \( W' \). However choosing
\[
W'(\omega) = \frac{\chi_{az}}{\Var\{ \hat{R}(\omega) \}} \tag{19}
\]
yields the minimum variance unbiased estimators (for a given \( \hat{R} \)) and the estimator variances become
\[
\Var\{ \hat{a} \} = \frac{\gamma_0 \chi_{ra} \chi_{az}}{\gamma_0 \gamma_2 - \gamma_1^2}, \quad \Var\{ \hat{b}' \} = \frac{\gamma_2 \chi_{ra} \chi_{az}}{\gamma_0 \gamma_2 - \gamma_1^2}, \tag{20}
\]
where
\[
\chi_{ra} = \sum_n \frac{|(\hat{h}_{ra} \otimes \hat{h}_{ra})_n|^2}{|\langle \hat{h}_{ra} \otimes \hat{h}_{ra} \rangle_0|^2}, \quad \chi_{az} = \sum_n \frac{|(\hat{h}_{az} \otimes \hat{h}_{az})_n|^2}{|\langle \hat{h}_{az} \otimes \hat{h}_{az} \rangle_0|^2}. \tag{21}
\]
The variance of the Fourier profiles are
\[
\Var\{ \hat{R}(\omega) \} = \chi_{az} \int d\varpi (a R(\omega) P(\varpi) + b)^2 W_R^2(\omega, \varpi), \tag{22}
\]
\[
\Var\{ \hat{P}(\varpi) \} = \chi_{ra} \int d\omega (a R(\omega) P(\varpi) + b)^2 W_P^2(\omega, \varpi). \tag{23}
\]
We can use the freedom of choice of weight functions (with the given constraints of eq. (11 and (12) to minimize the profile variance. However, since we at this stage (the initial stage), does not have a good model for $P$, and we need to satisfy the constrain of eq. (11) to obtain an unbiased estimate of the profile $R$, we choose $W_R \equiv 1$. Minimizing the variance of $\hat{P}$, yields

$$W_P(\omega, \varpi) = \frac{\lambda(\varpi) R(\omega)}{(a R(\omega) P(\varpi) + b)^2}; \quad \lambda(\varpi) \equiv \frac{1}{\int d\omega \frac{R^2(\omega)}{(a R(\omega) P(\varpi) + b)^2}}$$

and with this choice of weight function: $\text{Var}\{\hat{P}\} = \lambda \chi_{\text{ax}}$.

Since we need $a$, $b$ and $P$ to compute the variance used in the weight functions $W_P$ and $W'$, the procedure has to be done twice. The first time, flat weighting functions are used to get initial estimates $\hat{a}$ and $\hat{b}$ for $a$ and $b$, and as a model for $P$ we will use the lowest Fourier coefficients of $\hat{P}$. The procedure is then repeated with updated weight functions $W'$ and $W_P$ based on the latest estimated parameters.

**Remark I:** The size of the sub-images, the spacing between each of the estimation points $(\bar{t}, \bar{\tau})$ and the form of the windowing function $h$, used to compute the Fourier profiles, are defined such that the estimated values of $\varpi_{\text{dc}}$ and $a$ can be resampled to any grid later (they need to satisfy the Nyquist sampling criteria). This means that the estimation is done with at least 50% overlap in both range and azimuth direction. At the same time the estimation area in azimuth for TOPS mode must be sufficiently small to provide a continuous grid over the burst borders (restricted by the azimuth size of the overlap areas).
Remark II: The main contributor to the total time consumed by computing the Doppler centroid values is the computation of the 2D-Fourier profile represented by equation (2). A good estimate is that the algorithm will use slightly more than the time it takes performing 3 multiplications (window function, dechirping, complex conjugate) and one FFT of size \((256 \times 64)\) (range, azimuth) with 2.5 times overlap in both directions, covering the total data-set.

6.3.2 Computation of reference profiles

This subsection refers to Step II of the procedure outlined in Section 6.1.

Input parameters: Number of antenna elements in azimuth: \(N_a\), number of antenna elements in elevation: \(N_e\), antenna transmission (excitation) coefficients: \(Z_t\), antenna receive (excitation) coefficients: \(Z_r\), AUX_ECE file: multiplicative weighting (error) coefficients to the antenna transmission coefficients: \(\delta Z_t\), multiplicative weighting (error) coefficients to the antenna receiving coefficients: \(\delta Z_r\), radiation pattern of the T/R modules (embedded row pattern), \(f\), and LUT index.

Output parameters: Reference azimuth spectral profiles: \(P(\varpi; \bar{t})\)

Procedure: The one-dimensional (azimuthal) reference function in the two-dimensional data set space is computed as follows:

\[
P(\varpi; \bar{t}) = \left| \hat{A}(\varpi + \frac{\beta \gamma}{\gamma + \beta} \bar{\tau}; \frac{\beta}{\gamma + \beta} \bar{\tau} - \beta^{-1} \varpi) \right|^2
\]  

where \(\hat{A}(\varpi; \tau')\) is the two-way antenna diagram in the Fourier domain of the raw-data computed using equations (85) and (86). Here \(\bar{t} = (\bar{l}, \bar{\tau})\) is the center position of the estimation area, \(\gamma\) is the angular azimuth steering rate of the antenna and \(\beta\) is the angular signal Doppler rate given by equation (3). The reference profile is computed for each estimation position inside one burst and put into the \(REF\) array. The same \(REF\) array is to be used for several subsequent bursts. To speed up the computation, only a subset of the reference profile is computed in the range direction, and the profiles for the intermediate range positions are computed by interpolation.

6.3.3 Side-band correction

This subsection refers to Step III of the procedure outlined in Section 6.1.

Input parameters: 2D antenna model: \(\hat{A}\). Initial Doppler frequency: \(\varpi_{dc}\). Estimated signal energy: \(a\)

Output parameters: Side-band corrected estimated azimuth spectral profiles: \(\tilde{P}(\varpi; \bar{t})\)
Figure 4: Illustration of aliasing of energy from side-bands ($\ell = \pm 1$) into base-band. The vertical red lines define the base-band. The blue dotted lines are the doppler frequencies aliased into the base-band i.e. energy from nearest left and right neighboring areas mapped into estimation area. Here equal signal energy is assumed for the neighbor areas.

**Procedure:** The side-band correction is basically to correct the estimated azimuth Fourier profile for aliased energy arising from doppler frequencies outside the processed bandwidth or equivalently from neigbour areas left and right to the estimation area. If the signal energy is different in the neighbor areas, the uncorrected estimated profile will be skewed due to aliasing causing a bias in the estimated Doppler frequency. The correction of the estimated profile is done by combining the reference profile with the signal energy of the left and right neighbor areas. This is illustrated in Figure 4 assuming (for simplicity) equal signal energy in the left and right neighbor areas.

For the side-bands, we use the following approximation for the reference profile

$$P(\omega; t) \approx |\tilde{A}((1 + \frac{\gamma}{\beta})\omega + \frac{\beta\gamma}{\gamma+\beta} \tilde{\tau}; \frac{\beta}{\gamma+\beta} \tilde{\tau})|^2$$  \hspace{1cm} (26)

which only involves computing the azimuth raw-data Fourier diagram of the two-way antenna for only one time instance

$$\tau' = \frac{\beta}{\gamma + \beta} \tilde{\tau}$$

In order to avoid errors in the side-band correction, the reference function is centered around the initial Doppler offset. The energy of all critical sidebands (here indexed with $\ell$) is then computed as:

$$\alpha(\ell) P(\omega - \varsigma(\ell) + \ell\omega_\Delta; \tilde{t})$$  \hspace{1cm} (27)

This energy is subsequently removed from the observed Fourier profiles $\tilde{P}(\omega; \tilde{t})$ and the corrected profile is put back into the EST array. Here $2\omega_\Delta$ is the width of the base-band.
(bandwidth processed) given in rad/s. The corresponding energy $a^{(l)}$ for the side-bands and profile offsets

$$\varsigma^{(l)} = \frac{\beta}{\gamma + \beta} \omega_{dc}^{(l)}$$  \hspace{1cm} (28)

are computed at azimuth time position

$$\tilde{\tau}^{(l)} = \tilde{\tau} + \ell \beta^{-1} \omega_{\Delta}$$  \hspace{1cm} (29)

by combining the values of $a$ and $\omega_{dc}$ in the EST array. By critical side-bands, is meant the first side-bands ($\ell = \pm 1$) and the bands containing the first left and right side grating lobes. The grating-lobe correction is only needed for the estimation points having large steering angles.

6.3.4 Estimation of precision Doppler values:

This subsection refers to Step IV of the procedure outlined in Section 6.1.

**Input parameters:** Initial values of Doppler frequency: $\dot{\varsigma}$, Doppler spread: $\dot{\vartheta}$ and the corresponding variances on the estimation grid. Estimate of signal energy: $a$. Weight factors: $\chi_{ra}, \chi_{az}$. Estimated azimuth antenna pattern: $P(\varsigma)$. Modelled azimuth antenna pattern: $P(\varsigma)$.

**Output parameters:** Doppler frequency: $\omega_{dc}$, Doppler spread: $\vartheta$ and the corresponding variances on the estimation grid.

**Procedure:** The precision values of $a$, $\omega_{dc}$ and $\vartheta$ are computed by minimizing the following cost-function using standard minimization routine:

$$J(a, \varsigma, \vartheta') = \int d\omega W(\omega) \left| a P_{\vartheta'}(\omega - \varsigma) - \hat{P}(\omega) \right|^2$$  \hspace{1cm} (30)

where $P_{\vartheta'}$ is connected to the azimuth reference profile $P$ by

$$P_{\vartheta'}(\omega - \varsigma) \equiv \int d\varsigma' P(\varsigma') \frac{1}{\sqrt{2\pi} \vartheta'} e^{-\frac{1}{2} (\frac{\omega - \varsigma - \varsigma'}{\vartheta'})^2}$$  \hspace{1cm} (31)

and is introduced because there is a spread in the geophysical Doppler, mainly due to the orbital velocity of the ocean waves. The width $\vartheta'$ of the Doppler spread should be zero for land measurement and the Gaussian probability function collapses to a $\delta$-function. The minimizing of the cost function of eq. (30) is done iteratively by defining:

$$P(\omega) \equiv \left(1, \frac{\partial}{\partial \varsigma}, \frac{\partial}{\partial \vartheta'}\right) P_{\varsigma}(\omega - \varsigma), \quad \alpha \equiv (a, \Delta \varsigma, \Delta \vartheta')$$  \hspace{1cm} (32)
where $\Delta \varsigma$ and $\Delta \vartheta'$ represent the updates to the values of $\varsigma$ and $\vartheta'$ from the previous iteration. The solution of one iteration is given by

$$\hat{\alpha} = \int d\varpi \, \Gamma^{-1} \cdot P(\varpi) \, \tilde{P}(\varpi) \, W(\varpi)$$  \hspace{1cm} (33)

where $\Gamma$ is a $3 \times 3$-matrix with elements

$$\gamma_{m,n} = \int d\varpi \, P^{(m)}(\varpi) \, P^{(n)}(\varpi) \, W(\varpi)$$  \hspace{1cm} (34)

Here $P^{(\ell)}$ represents the $\ell$-th element of $P$. The covariance matrix of this estimator is given by

$$\text{Covar}\{\hat{a}\} = \chi_{\text{az}} \int d\varpi \, (\Gamma^{-1} \cdot P(\varpi))^T (\Gamma^{-1} \cdot P(\varpi)) \, \text{Var}\{\tilde{P}(\varpi)\} \, W^2(\varpi)$$  \hspace{1cm} (35)

The variance of the estimator $\alpha$ is minimized by choosing

$$W(\varpi) = \frac{\chi_{\text{ra}}}{\text{Var}\{\tilde{P}(\varpi)\}} ,$$  \hspace{1cm} (36)

and the covariance matrix simply becomes

$$\text{Covar}\{\hat{a}\} = \chi_{\text{ra}} \chi_{\text{az}} \, \Gamma^{-1} .$$  \hspace{1cm} (37)

where $\chi_{\text{ra}}$ and $\chi_{\text{az}}$ are computed in Step I. After the final minimization the updated values of $a$ and

$$\varpi_{dc} = (1 + \frac{1}{2}) (\varsigma - \varsigma_0) \hspace{1cm} (38)$$

$$\vartheta = (1 + \frac{1}{2}) \vartheta' \hspace{1cm} (39)$$

are stored in the EST array, where $\varsigma_0$ is the offset from zero of the reference profile corresponding to zero azimuth steering angle of the antenna. The algorithm will not be correcting for absolute errors predicted by the computed antenna diagrams, but only relative errors. This means that the estimated Doppler centroid is still going to be the sum of the geophysical Doppler and the effects due to errors in the antenna model.

### 6.3.5 Resampling to output grid and estimation of radial velocity

This subsection refers to Step V of the procedure outlined in Section 6.1.

**Input parameters:** Doppler frequency: $\varpi_{dc}$, Doppler spread: $\vartheta$, Radial velocity: $U_r$ and the corresponding variances on the input grid. Geometric Doppler offset: $\varpi_{dc}$. L2 Aux PP2 file. AUX WND: ECWMF wind field. AUX WAV: WaveWatch III. AUX CLM: Coastline data.
**Output parameters:**  Doppler frequency: $\omega_{dc}$, Doppler spread: $\vartheta$, Radial velocity: $U_r$ and the corresponding variances on the output grid.

**Procedure:**  By combining the EST-array (for TOPS from all swaths), the Doppler centroid $\omega_{dc}$ and width of the Doppler spread $\vartheta$ are resampled to the output grid, and the radial velocity is computed from the relation:

$$U_r = \frac{\omega_{dc} - \tilde{\omega}_{dc}}{2k_r}$$  \hspace{1cm} (40)

where $\tilde{\omega}_{dc}$ is the known (from attitude data) geometric Doppler offset. Those three values (radial velocity, Doppler offset, Doppler spread) are in combination with the variance of the estimates, stored in the structure RET array and returned to the caller. The size of the output grid is computed from the size of the EST-array ($M_{est}, N_{est}$), the input SLC pixel sizes, the percentage of estimation block overlap (50%), and the output grid resolutions ($RaRes, AzRes$). Internal IDL resampling routine will be used for the resampling operation. Then the percentage land overlap is computed for each output grid using the lat/lon of the grid point and the AUX CLM file (similar as for OSW).

Finally, the RVL output data structure is generated including resampling of auxiliary data (ECMWF, Wavewatch III (optional) into the output grid, and stored internally for later input to the OCN merging process.
7 Input Files

The Sentinel-1 RVL processing will access at input both SAR products and external data. These are specified in the following subsections, and a detailed description of these products and data can be found in [A-4] and [A-3], respectively.

7.1 SAR Image Products

The RVL processing system can access and process the SAR SLC data products listed below. The SLC products supported are the internal L1 SLC product suitable for L2 processing.

- Sentinel-1 Stripmap Mode (SM) on S-1 format - SM SLC (internal)
- Sentinel-1 Wave Mode (WV) on S-1 format - WV SLC (internal)
- Sentinel-1 Interferometric Wide-swath Mode (IW) on S-1 format - IW SLC (internal)
- Extra-Wide-swath Mode (EW) - EW SLC (internal)
- ASAR L1 IMS and WVI products, supported in test mode only

7.1.1 SAR Product Annotations

The RVL processing algorithm will extract the following key data (parameters) from the SLC (internal) product:

- Range and azimuth dimensions of input SLC image
- Range and azimuth pixel size of input SLC image
- Time of first line in SLC data
- Two way slant range time to first pixel in SLC data
- State vector (at least one)
- Number of bursts in input SLC (for TOPS)
- Raw data azimuth time (for each burst for TOPS)
- Raw data two way slant range time (for each burst for TOPS)
- Rawdata PRF and sampling rate (for each burst for TOPS)
- Size of raw data (for each burst for TOPS)
- Steering rate (for TOPS data)
- Yaw, pitch and roll of satellite platform at zero doppler time
- Doppler centroid frequency as predicted from orbit and attitude data
- Chirp replica

7.2 Auxiliary Data

This section is an overview of the auxiliary files used by RVL processing, and more information is provided in the Auxiliary Specification document [A-3], the Product Specification document [A-4], and Interface Control document [A-12]. The auxiliary file description is here separated into internal and external files. The internal files are meant not to be changed during mission, while the external files may change. The auxiliary data from ECMWF and Wavewatch III are not explicitly used by the algorithm but provided as part of the RVL component of the OCN product for the benefit of users. Detailed specifications of the auxiliary data can be found in [A-3].

7.3 Internal Auxiliary Data Files

7.3.1 Coastline and Land Masking Data - LOP_CLM

Doppler and RVL processing is not performed if land coverage is greater that 10% in the imagette considered. The land coverage is estimated as the ratio between the surface area of imagette and the surface area of a local land mask that covers the imagette. The coastline to be used should be quite accurate; an accurate shoreline polyline from the GSHHS is required. This shoreline database is available at


7.3.2 S-1 Antenna Embedded Row Pattern - LOP_PAT

The embedded row patterns are required by the S-1 IPF as part of the L2 RVL component. The data is used to derive accurate Doppler estimation by synthesizing a radiation beam pattern using an idealized antenna model.

7.3.3 S-1 Antenna Excitation Coefficients - LOP_COE

The excitation coefficients are required by the S-1 IPF as part of the L2 RVL component. The data is used to derive accurate Doppler estimation by synthesizing a radiation beam pattern using an idealized antenna model.
7.3.4 S-1 Antenna LUT - LOP_LUT

LUT index - the association of the excitation coefficients with the PRI within a burst. The data is used in synthesizing a radiation beam pattern using an idealized antenna model.

7.3.5 Internal Processing Parameter File - PRM_LOPIn

When the L2 processor receives a job order request, the L2 processor will access the processing parameters via the L2 Processor Parameter Auxiliary File (AUX PP2 file). There is also an option to include an IPF internal parameter file for the L2 RVL processor in case more parameters than those defined in the AUX PP2 file. These parameters will be provided in the Internal Processing Parameter File, and will be used during the algorithm development phase for tuning the L2 RVL processor performance.

7.4 External Auxiliary Data Files

7.4.1 S-1 Antenna Error Matrix Auxiliary Data - AUX_ECE

The error matrix (EM) is required by the S-1 IPF as part of the L2 RVL component. The EM is used to derive accurate Doppler estimation by synthesizing a radiation beam pattern using an idealized antenna model. The error matrix is used to correct the t/r excitation coefficient with the latest available correction data. This file is optional.

7.4.2 Atmospheric Model Wind Field - AUX_WND

Wind speed and direction at 10 m above the sea surface from the ECMWF atmospheric model is required with spatial and temporal resolution of at 0.125x0.125 degrees every 3 hours. The model wind field will be resampled to RVL grid and provided as part of the RVL product output for the benefit of users.

7.4.3 Wavewatch III Model - AUX_WAV

The Wavewatch III Stokes drift is a vector calculated as the third order moment of the wave spectra. The Stokes drift is mostly dependent on the wind sea which is not imaged spectrally by the SAR. Stokes drift forecast files in NetCDF format generated by the operational Wavewatch III model run at IFREMER will be used. The Wavewatch III model provides surface Stokes drift velocity and direction for frequency range $f_{\text{min}} = 0.032$ to $f_{\text{max}} = 0.032$ Hz with a spatial resolution of at least 0.5 degrees every 6 hours. The Stoke drift field will be resampled in to RVL grid and provided as part of the RVL product output for the benefit of users.
7.4.4 L2 Processor Parameter Auxiliary Data - AUX_PP2

The RVL processing is flexible, and the processing can be configured with the key parameters listed below.

- Size of Doppler estimation block in range [m]
- Size of Doppler estimation block in azimuth [m]
- Size of grid cell interval in range direction [m]
- Size of grid cell interval in azimuth direction [m]
8 Symbols

The following list provides the definitions of most important symbols used in the algorithm description.

$t = (t, \tau)$: 2D time variable of SLC image (range, azimuth)

$T$: Scatter two-way time

$A$: 2D angular antenna pattern for stiff antenna

$\tilde{A}$: 2D angular pattern for phase array antenna

$\Theta_0$: Antenna elevation angle

$\Theta$: Local elevation angle to scatterer

$\Phi_0$: Antenna squint angle

$\dot{\Phi}_0$: Antenna squint rate

$\beta$: Signal Doppler rate

$\gamma$: Angular frequency squint rate

$v_r$: Effective radar velocity

$v_a$: Azimuth velocity of zero-Doppler plane

$v_p$: Platform velocity

$\omega = (\omega, \varpi)$: 2D-Fourier SLC image variable (range, azimuth)

$Z^t$: Array containing the corrected antenna transmission coefficients.

$Z^r$: Array containing the corrected antenna receive coefficients.

$f$: Array containing the radiation pattern of the T/R modules (embedded sub-array pattern).

$\hat{\omega}_{dc}$: Array containing the error in Doppler centroid values as function of elevation angle

$I_c$: Complex SAR image (internal L1 SLC product)

$k_r$: Radar wavenumber

$\omega_o$: Radar carrier frequency (angular)

$\varpi_\Delta$: Bandwidth of SLC in azimuth
\( \hat{P} \): Array containing the observed azimuth Fourier profile
\( P \): Array containing the reference azimuth Fourier profiles.
\( R \): Array containing the reference range Fourier profiles.
\( M_{\text{est}} \): Number of estimation points in range direction (per burst for TOPS)
\( N_{\text{est}} \): Number of estimation points in azimuth direction (per burst for TOPS)
\( N \): Number of points in azimuth Fourier profiles
\( W \): Weight function used in the cost function
\( h \): Weight function used in profile estimation
\( (\bar{t}, \bar{\tau}) \): 2D time position of estimation points (range, azimuth)
\( a \): Scalar containing the estimated energy of the signal.
\( b \): Scalar containing the estimated energy of the additive noise in the signal.
\( \varpi_{\text{dc}} \): Scalar containing the estimated angular Doppler frequency offset.
\( \vartheta \): Scalar containing the estimated width of the Doppler frequency spread.
\( U_r \): Scalar containing the estimate of radial velocity.
\( \sigma^2_{\varpi_{\text{dc}}} \): Scalar containing the variance of the estimate of \( \varpi_{\text{dc}} \).
\( \sigma^2_{\vartheta} \): Scalar containing the variance of the estimate of \( \vartheta \).
\( \sigma^2_{U_r} \): Scalar containing the variance of the estimate of \( U_r \).
\( s_{\text{rep}}(t) \): Pulse replica
\( L^a \): Size of antenna in azimuth direction
\( L^e \): Size of antenna in elevation direction
9 Output Product Content

The following is a brief description/content of the numerical NetCDF file for the RVL component of the OCN product. When the OSW, RVL and OWI components are generated on internal files the final OCN product will be generated by merging the information from these files into a common NetCDF file.

The L2 RVL annotations data set includes the following dimensions:

- Number of RVL cells in the range direction
- Number of RVL cells in the azimuth direction

The L2 RVL data set also includes the following variables per RVL cell:

- Geodetic latitude at cell centre [deg]
- Geodetic longitude at cell centre [deg]
- Zero Doppler time at cell centre
- Two way slant range time at cell centre
- Ground range size of estimation area [m]
- Azimuth size of estimation area [m]
- Estimated Doppler frequency at cell centre [Hz]
- Estimated standard deviation of Doppler frequency at cell centre [Hz]
- Estimated width of Doppler frequency spectra at cell centre [Hz]
- Estimated standard deviation of width of Doppler frequency spectra at cell centre [Hz]
- Estimated radial velocity at cell centre [m/s]
- Estimated standard deviation of radial velocity at cell centre [m/s]
- Predicted geometric Doppler frequency (from Level 1 resampled to L2 doppler grid) [Hz]
- Yaw of satellite platform at zero Doppler time (from Level 1)
- Pitch of satellite platform at zero Doppler time (from Level 1)
- Roll of satellite platform at zero Doppler time (from Level 1)
- Local northing angle [degN]
• Incidence angle at cell centre [deg]
• Signal-to-noise ratio [dB]
• Confidence of Doppler frequency estimate
• Auxiliary Data derived statistics
  – Percentage of land coverage for each cell
  – Land flag
  – Ice flag
  – Doppler frequency from antenna mispointing at cell centre [Hz]
  – ECMWF wind speed [m/s]
  – ECMWF wind direction [degN]
  – Radial component of surface stokes drift from Wavewatch III model [m/s]
    (optional)
  – Azimuthal component of surface stokes drift from Wavewatch III model [m/s]
    (optional)
  – Sea state induced radial velocity from model [m/s] (optional)

The L2 RVL component of the OCN product includes the global attributes:

• Mission name
• Level1 source filename
• Processing time
• Polarization
• State vector
• Doppler algorithm version
• Grid cell size [$m^2$]
A Background theory

This section outlines some background theory for the Doppler estimation algorithm described in the subsequent section. In sub-section A.1 basic notations, variables and coordinate system (Figure 5) are defined, and relation between complex SAR image and raw data is derived in frequency domain. In sub-section A.2 the azimuth time-frequency relations for phase array antenna are derived.

![Curved track radar-to-target geometry](image)

Figure 5: Curved track radar-to-target geometry. \( p \) is the function mapping from 3D zero-Doppler system to \( xyz \) coordinate system, \( t \) is the fast time, \( \tau \) is the slow time, \( \Theta \) is the elevation angle, \( \Phi \) is the squint angle.

A.1 SAR imaging process

A.1.1 SAR raw data

The base-band sampled signal reflected from a frozen surface, can be written as a two-dimensional integral over the scatter location \( t = (t, \tau) \) in the 2D zero-Doppler coordinate system:

\[
I_{\text{raw}}(t') = \int dt \ e^{-i\omega_0 T(t' - \tau, t)} \gamma_c(t) s(t' - T(t' - \tau; t)) V_0(t' - \tau; t)
\]  

(41)

where \( \gamma_c \) is the complex back-scatter coefficient of the imaged surface, \( s \) is the modulation of the transmitted signal, \( V_0 \) is the antenna ground pattern, and \( T \) is the two-way time given as

\[
T(t' - \tau; t) = \sqrt{t'^2 + \nu^2(t)(t' - \tau)^2}, \quad \nu(t) = \frac{2\nu}{c},
\]  

(42)
where \( v_r \) is the effective radar velocity.

The range direction Fourier transform of the raw data is

\[
\hat{I}_{\text{raw}}(\omega, \tau') = \hat{s}(\omega) \int dt \, e^{-i(\omega_0 + \omega)T(\tau' - \tau; t)} \, \gamma_c(t) \, V_0(\tau' - \tau; t),
\]

where \( \hat{s} \) is the Fourier transform of \( s \). In this domain we observe that SAR imaging process is independent of the form of the modulated signal. Strictly speaking, this equation is the correct starting point for definition of the raw data, since the antenna pattern is also a function of range frequency \( \omega \), which can only be neglected for narrow-band signals.

### A.1.2 SAR raw data Fourier domain

The 2D Fourier transform of the raw data can be written as

\[
\hat{I}_{\text{raw}}(\omega) = \hat{s}(\omega) \int dt \, \gamma_c(t) \, \frac{1}{2\pi} \int d\tau' \, e^{-i\phi(\tau')} \, V_0(\tau' - \tau; t),
\]

where \( \omega = (\omega, \omega) \) is the 2D-Fourier variable and the phase function in the last integral is defined by

\[
\phi(\tau') = \omega \tau' + (\omega_0 + \omega) T(\tau' - \tau; t).
\]

To find an explicit expression for this integral, we will use the method of stationary phase. By expanding the phase to order two around the stationary phase value \( \tau'_s \)

\[
\phi(\tau') \approx \phi(\tau'_s) + \dot{\phi}(\tau'_s)(\tau' - \tau'_s) + \frac{1}{2} \ddot{\phi}(\tau'_s)(\tau' - \tau'_s)^2,
\]

and demanding that \( \dot{\phi}(\tau'_s) = 0 \), gives the following solution for the stationary phase value

\[
\tau'_s = \tau - \frac{\omega_0}{\nu^2(t) \Omega(\omega, t)},
\]

where

\[
\Omega(\omega, t) \equiv \sqrt{(\omega_0 + \omega)^2 - \omega^2 / \nu^2(t)}.
\]

By inserting the expression for the stationary phase value \( \tau'_s \), and computing the integral, the 2D Fourier transform of the raw data can be approximated by:

\[
\hat{I}_{\text{raw}}(\omega) \approx \hat{s}(\omega) \int dt \, e^{-i\omega_0 T - i\Omega(\omega, t)t} \gamma_c(t) \, V(\omega, t),
\]

where

\[
V(\omega; t) = V_0(-\frac{\omega}{\nu^2(t) \Omega(\omega, t)}; t) \sqrt{\frac{(\omega_0 + \omega)^2 t}{2\pi \nu^2(t) \Omega^2(\omega, t)}}
\]

is the azimuth direction envelope function of the Fourier domain. This is a slowly varying function of the scatter location \( t \). The azimuth direction envelope function is directly related to the two-way antenna pattern (see Eq. 61).
A.1.3 SAR data focusing

Without loss of generality, we may write the Fourier transform of the transmitted signal as

$$\hat{s}(\omega) = U(\omega) e^{-i\psi(\omega)}$$  \hspace{1cm} (51)

where $U$ is the magnitude of $\hat{s}$. Further we define:

$$\Psi(\omega; t) \equiv (\Omega(\omega, t) - (\omega_0 + \omega)) t + \psi(\omega) + \omega \cdot \bar{t}$$  \hspace{1cm} (52)

where $\bar{t}$ is some reference time position. With those definitions, the Fourier transformed SAR raw data can be written as

$$\hat{I}_{\text{raw}}(\omega) = U(\omega) \int dt \, e^{-i\omega(t - \bar{t}) - i\Psi(\omega; t)} \gamma_c(t) e^{-i\omega_0 t} V(\omega; t)$$  \hspace{1cm} (53)

Here, $e^{-i\Psi(\omega; t)}$ represents the transfer function between the complex focused SAR data and the raw data. Since there exists an area around $t = \bar{t}$ where $|\Psi(\omega; \bar{t}) - \Psi(\omega; \bar{t})| < \epsilon$, for any real positive $\epsilon > 0$, the phrase Exact Transfer Function is meaningful, and the SAR data may be focused with $e^{i\Psi(\omega; \bar{t})}$ as a reconstruction transfer function

$$\forall \; \epsilon > 0 \; \exists \; \delta > 0 : \left| I_c(t') - \int d\omega \, e^{i\omega(t' - \bar{t})} e^{i\Psi(\omega; \bar{t})} \hat{I}_{\text{raw}}(\omega) \right| < \epsilon \; \text{for} \; |t' - \bar{t}| < \delta,$$  \hspace{1cm} (54)

where $I_c$ is the theoretical perfectly focused complex SAR data. The practical problem of using $\Psi(\omega; \bar{t})$ as the phase function of the reconstruction transfer function is that the area around $t = \bar{t}$ where satisfying results are achieved is small.

A better approach is to use the following expansion for the weak two-way time dependence in the phase function

$$\Psi(\omega; t) \approx \Psi(\omega; \bar{t}) + \Theta(0, \omega, \bar{t}, \bar{t}) - \Psi(0, \omega, \bar{t}, \bar{t}) + (t - \bar{t}) \omega \frac{\partial^2 \Psi}{\partial \omega^2} (0, \omega, \bar{t}, \bar{t}) \bar{t}.$$  \hspace{1cm} (55)

The focusing scheme then becomes

$$I_c(t') = \int d\omega \, e^{i\omega(t' - \bar{t})} e^{i\Psi(0, \omega)} \int d\omega \, e^{i\omega(t' - \bar{t})} e^{i\Psi(0, \omega)} \hat{I}_{\text{raw}}(\omega),$$  \hspace{1cm} (56)

yielding the following relation between the focused SAR data and the backscatter coefficient

$$I_c(t') = \int dt \, H(t' - t; t) \gamma_c(t) e^{-i\omega_0 t},$$  \hspace{1cm} (57)

where the impulse response function is given by:

$$H(t''; t) \approx \int d\omega \, e^{i\omega t''} \int d\omega U(\omega) V(\omega; t).$$  \hspace{1cm} (58)
To illustrate the effects of the different terms of the phase function, we approximate $\Psi_1$ to first order in $t$:

$$
\Psi_1(t, \omega) \approx (t - \bar{t}) \frac{\partial \phi}{\partial t}(0, \omega, \bar{t}, \bar{\tau}) \equiv (t - \bar{t}) \mu(\bar{\omega}).
$$

(59)

With this approximation, the relation between the Fourier domains of the raw and the compressed SAR data can be expressed as:

$$
\hat{I}_c(\omega, \bar{\omega}) \approx e^{i\Psi_0(\omega, \bar{\omega})} \hat{I}_{\text{raw}}(\alpha(\bar{\omega})\omega + \mu(\bar{\omega}), \bar{\omega}).
$$

(60)

This means that the $\Psi_1$-term in (55) is causing a range frequency shift (through $\mu$), while the term containing $\alpha$ is causing a scaling of the the range frequency domain. This is often referred to in the literature as Stolt migration.
A.2 Azimuth time-frequency relations

In this section we start describing how the antenna ground pattern is related to the 2D angular antenna diagram of the radar, then how the azimuth direction frequency band is limited by the antenna diagram, and how the antenna squint angle is shifting the location of the frequency band. In sub-section A.2.3, estimation of the Doppler centroid is discussed, for the raw and focused data cases, respectively. In sub-section A.2.1 to A.2.3 is the theory given for a stiff rotated antenna, whereas in the last sub-section A.2.4, the same properties are discussed, using a phased array antenna.

A.2.1 Antenna pattern

The antenna ground pattern can be written as

\[ V_0(\tau' - \tau; t) = \frac{C}{\tau_0(\tau' - \tau; t)} A(\Phi(\tau' - \tau; t) - \Phi_0(\tau'), \Theta(t) - \Theta_0), \]  

(61)

where \( A \) represents the 2D angular antenna pattern, where \( \Theta_0 \) is the antenna elevation angle, \( \Theta \) is the local elevation angle to the target, \( \Phi_0 \) is the antenna squint angle and\n
\[ \Phi(\tau' - \tau; t) \approx \frac{2\nu_a(t)(\tau' - \tau)}{c^2} \]  

(62)

represents the angle between the zero-Doppler direction at time \( \tau' \) and the direction to a point on the ground in time position \( t \), see Figure 5. The angular antenna pattern is generally also a function of the signal frequency and antenna pitch angle, but explicit notation of those dependencies are omitted for simplicity. \( \nu_a \) is the azimuth velocity of zero-Doppler plane.

A.2.2 Frequency band location

Assume that the antenna squint angle is changing through the imaging burst. If the change is linear in time, we may write the antenna squint angle as

\[ \Phi_0(\tau') = \Phi_0(\tau'_0) + \dot{\Phi}_0(\tau'_0)(\tau' - \tau'_0) \]  

(63)

where \( \dot{\Phi}_0 \) is the antenna squint rate. In the following, we will, without loss of generality, assume that the azimuth time \( \tau' \) of the raw data is zero for the center position of the raw data burst (\( \tau_0 = 0 \)). By combining this expression with the time-frequency relation

\[ \tau' - \tau = -\beta^{-1} \omega, \]  

(64)

where

\[ \beta = \frac{\nu^2(\omega_0 + \omega)}{T} = \frac{\nu^2 \Omega}{l} \]  

(65)

is the signal Doppler rate (in rad/s²). Substituting for \( \tau \) or \( \tau' \), respectively, we get the following time-frequency relations for raw and focused data:
**Raw data time-frequency relation:** Let $\tau \rightarrow \tau + \beta^{-1} \omega$, the expression for the antenna diagram becomes

$$A(\Phi - \Phi_0, \cdot) \longrightarrow A\left( -\frac{c}{2v_p \Omega} \left( \omega - \omega_{dc} - \gamma \tau \right), \cdot \right)$$

(66)

where

$$\gamma = -\frac{2v_p \Omega}{c} \Phi_0$$

(67)

is the angular frequency squint rate of the antenna and

$$\omega_{dc} = -\frac{2v_p \Omega}{c} \Phi_0$$

(68)

is the angular Doppler centroid offset frequency. Here $v_p$ is the platform velocity and $\Omega$ is given by (48). For the center of the burst, where $\tau^i$ is zero, we get the stripmap mode ($\gamma = 0$) azimuth frequency envelope function centered around $\omega_{dc}$, but generally, the expression is now centered around $\omega_{dc} + \gamma \tau^i$, meaning that the azimuth frequency band is moving as function of azimuth time. This effect is illustrated in time-frequency diagram of Figure 6-a. The anti-diagonal lines, with slope equal to $-\beta$, represents the influence of equally spaced targets. Note that we have assumed a Doppler rate equal to the pulse repetition frequency (PRF) in this plot.

**Focused data time-frequency relation:** By letting $\tau^i \rightarrow \tau - \beta^{-1} \omega$, the expression for the antenna diagram becomes

$$A(\Phi - \Phi_0, \cdot) \longrightarrow A\left( -\frac{c}{2v_p \Omega} \left( 1 + \frac{\gamma}{\beta} \right) \left( \omega - \frac{\beta}{\beta + \gamma} \left( \omega_{dc} + \gamma \tau \right) \right), \cdot \right)$$

(69)

This expression is different from the usual Fourier domain envelope function for stripmap mode focused data ($\gamma = 0$) in two ways: First, the bandwidth is changed with a factor $\beta/(\beta + \gamma)$, and second, it is not centered around the angular Doppler centroid offset frequency (not even for $\tau = 0$), but around $\beta/(\beta + \gamma)(\omega_{dc} + \gamma \tau)$. This means that the estimation procedure for the Doppler centroid offset anomaly is slightly more complicated, since $\beta$ is a function of the estimation position. The time-frequency diagram of the focused data is illustrated in Figure 6-b. Here we observe that the lines separating the colors in the time-frequency diagram are vertical, meaning that the targets are sharply located in time.

**A.2.3 Doppler centroid estimation**

By deramping the raw data or the focused data with chirps aligning the frequency bands, the azimuth direction Fourier domain envelope functions becomes proportional to

**Raw:**

$$A\left( -\frac{c}{2v_p \Omega} \left( \omega - \omega_{DC} \right), \cdot \right)$$

(deramped with $e^{\frac{i}{2} \gamma \tau^2}$) 

(70)

**Focused:**

$$A\left( -\frac{c}{2v_p \Omega} \left( 1 + \frac{\gamma}{\beta} \right) \left( \omega - \frac{\beta \omega_{DC}}{\beta + \gamma} \right), \cdot \right)$$

(deramped with $e^{\frac{i}{2} \frac{\beta^2 \gamma^2}{\beta + \gamma} \tau^2}$) 

(71)
The Doppler centroid frequency offset $\omega_{DC}$ can now be found, by estimating the location of the envelope maximum. The value is equal the location of the envelope maximum for the raw data case and equal to $1 + \gamma / \beta$ times the envelope maximum for the focused data case. Here, ordinary stripmap mode centroid estimators may be used.

### A.2.4 Phased array antennas

So far, only the ideal stiff rotated antenna case has been discussed, in this sub-section we will introduce the phased array antenna. The general form of the transmission or receive pattern of a regular gridded 2D-antenna with $N \times M$ elements, can be written as

$$A^{t/r}(\zeta; \zeta_0) = \sum_{n,m} \tilde{Z}^{t/r}_{n,m}(\zeta_0) e^{i\zeta \cdot (nN, mM)} f_{n,m}(\zeta)$$

(72)

where $f_{n,m}$ represents the pattern of each individual module, $\tilde{Z}^{t/r}_{n,m}$ is the receive/transmit excitation coefficients, and $\zeta$ and $\zeta_0$ is the signal- and antenna steering direction, related to the azimuth and elevation angles in the following way

$$\zeta = (\zeta, \xi) \equiv (k_r L^a \sin \Phi, k_r L^e \sin \Theta) .$$

(73)

Here is $L^a$ and $L^e$ the size of the total antenna in azimuth and elevation direction, respectively. The two-way pattern is given by

$$A(\zeta; \zeta_0) = A^t(\zeta; \zeta_0) A^r(\zeta; \zeta_0) .$$

(74)

**Beam formation:** The big advantage of the phased antenna, is not only the ability to steer the antenna by adding a phase shift to the T/R coefficients

$$\tilde{Z}^{t/r}_{n,m}(\zeta_0) \equiv \tilde{Z}^{t/r}_{n,m}(\zeta_0) e^{-i\zeta_0 \cdot (nN, mM)}$$

(75)
Figure 7: Antenna pattern showing the difference between a classic antenna (red) where all T/R coefficients $Z$ are equal to one, and an antenna where the receive coefficients are formed to reduce the energy in the side-lobes (black) by steering the zeros away from the zeros of the transmitted pattern.

but also have the freedom to form the transmitting and receive antenna diagrams individually. By shaping the two diagrams such that the zero-crossing is not co-located, the energy of the side-lobes of the two-way antenna diagram is dramatically reduced. In the following discussion, we drop the explicit notation for the elevation directions on the left side, and rewrite equation (72) as

$$A^{tr}_{\frac{\zeta}{r}}(\zeta; \zeta_0) = \sum_{n,m} Z_{r,t}^{n,m}(\zeta_0) e^{i(\zeta-\zeta_0)(\frac{\pi}{N}, \frac{\pi}{M})} f_{n,m}(\zeta)$$

(76)

The big challenge, in the beam formation for TOPS systems, is to be able to keep the shape of the main lobe of the antenna constant for different azimuth steering directions:

$$A(\zeta; \zeta_0) \approx A(\zeta - \zeta_0; 0), \quad \zeta \in (\zeta_0 - \frac{\Delta \zeta}{2}, \zeta_0 + \frac{\Delta \zeta}{2})$$

(77)

where $\Delta \zeta$ is the width of the main-lobe, and at the same time keep the energy in the side-lobes low. The disadvantage of using a phased array is the appearance of the grating lobes. All those effects are visualized easily with some small simplifications of the model.

Simplifications: By assuming all $f_{n,m}$ to be the radiation diagrams of perfect rectangular sub-antennas with the same size ($L^a/N \times L^e/M$):

$$f_{n,m}(\zeta) \approx f^a(\zeta) f^e(\xi) = \text{sinc} \frac{\zeta}{2N} \text{sinc} \frac{\xi}{2M},$$

(78)
then the antenna diagrams may be written as

\[ A^{1/r}(\zeta; \zeta_0) \approx f^a(\zeta) \sum_n z_n^{1/r}(\zeta_0) e^{i(\zeta-\zeta_0)n/N}, \]  

(79)

where

\[ z_n^{1/r}(\zeta_0) \equiv f^e(\xi) \sum_m Z_n^{1/r}(\zeta_0, \xi_0) e^{i(\xi-\xi_0)m/M}. \]  

(80)

The two-way antenna diagram can with this simplification be written as

\[ A(\zeta; \zeta_0) \approx \left\{ f^a(\zeta) \right\}^2 \sum_\ell c_\ell(\zeta_0) e^{i(\zeta-\zeta_0)\ell/N}, \]  

(81)

where

\[ c_\ell(\zeta_0) \equiv \sum_n z_{\ell-n}^r(\zeta_0)z_n^r(\zeta_0) \]  

(82)

is different from zero for \((2N - 1)\) values of \(\ell\). This means there exist \(2N - 1\) degree of freedom to keep the main-lobe of the diagram close to constant and at the same time minimizing the energy of the side-lobes. To keep the beam-form close to constant, for different steering direction, is the same as the integrated effects of \(c_\ell\) mimics

\[ c_\ell(\zeta_0) \sim c_\ell(0) \left\{ \frac{f^a(\zeta-\zeta_0)}{f^a(\zeta)} \right\}^2. \]  

44
In Figure 7 is the steering angle equal to zero, and we observe how, the side-lobes are reduced by the effect of not having the same locations of the zero-crossing in the receive diagram as in the transmitted one. In Figure 8 the steering angle is not zero, and we observe how the grating-lobes grow as the main lobes of the periodic beam-train are shifted away from the zeros of \( f^a(\zeta) \).

**Time-frequency relations:** Given a raw-data time \( \tau' \) the azimuth direction component \( \zeta \) of the directional vector is related to a target in zero Doppler time position \( t \) as follows

\[
\zeta = k_1 L^a \frac{2v_a(t)}{c_f} (\tau' - \tau) = -\frac{(\omega + \omega_0)L^a}{2\Omega v_p} \varpi \equiv \varrho \varpi
\]  
(83)

where the second relation is coming from using the time-frequency relation of equation (64). \( L^a \) is length of antenna in azimuth. For the steering direction we have

\[
\zeta_0 = \varrho \gamma \tau'.
\]  
(84)

Combining those two equations, the antenna diagrams becomes

\[
\hat{A}(\varpi; \tau') = \hat{A}'(\varpi; \tau') \hat{A}'(\varpi; \tau')
\]  
(85)

\[
\hat{A}'(\varpi; \tau') \equiv A'(\varpi; \varrho \gamma \tau') = \sum_{n,m} Z_{n,m}'(\tau') \delta Z_{n,m}' e^{i(\varpi - \gamma \tau')\theta + i(\xi - \xi_0)\frac{n}{M} f_{n,m}(\varpi, \xi)},
\]  
(86)

where we have chosen to annotate the coefficients as function of azimuth time \( \tau' \). Here \( \delta Z_{n,m}' \) are the multiplicative weighting (error) coefficients to the \( t/r \) modules provided as auxiliary data. In the following we will use the following substitution \( \tau' \rightarrow \tau - \beta^{-1} \varpi \) to express the focused time-frequency relations. Assume we have a frequency offset \( \varpi_{dc} \) the two-way antenna diagram becomes:

**Raw:**

\[
\hat{A}(\varpi - \varpi_{dc}; \tau')
\]  
(87)

**Focused:**

\[
\hat{A}(\varpi - \varpi_{dc}; \tau - \beta^{-1} \varpi)
\]  
(88)

and if deramped, we get:

**Raw:**

\[
\hat{A}(\varpi - \varpi_{dc} + \gamma \tau'; \tau') \quad \text{(deramped with } e^{i \frac{\varrho}{\gamma} \tau'^2})
\]  
(89)

**Focused:**

\[
\hat{A}(\varpi - \varpi_{dc} + \frac{\beta}{\gamma + \beta} \tau; \tau - \beta^{-1} \varpi) \quad \text{(deramped with } e^{i \frac{\beta}{\gamma + \beta} \tau'^2})
\]  
(90)

If (and only if) the form of the main-lobe is invariant of the azimuth steering direction, we then get the following results:

**Raw:**

\[
\hat{A}(\varpi - \varpi_{dc}; 0) \quad \text{(deramped with } e^{i \frac{\varrho}{\gamma} \tau'^2})
\]  
(91)

**Focused:**

\[
\hat{A}((1 + \frac{\beta}{\gamma})(\varpi - \frac{\varpi_{dc}}{\beta + \gamma}); 0) \quad \text{(deramped with } e^{i \frac{\beta}{\gamma + \beta} \tau'^2})
\]  
(92)

This result is similar to the result of the previous sub-section. The requirements of having a time invariant shape of the main-lobe is a strong requirement. Even, if the antenna form is created such that the energy and the mean location (Doppler offset) is invariant of time \( \tau' \), the antenna diagram (in the focused Fourier-domain) will not necessarily be radiometric or mean location invariant of time \( \tau \).
B Expected Estimation Quality Performance

This section derives the expected estimation quality performance for the ideal case of no errors in the antenna.

Assuming linear statistics:

\[
\hat{P}(j) - \hat{P}(j) \approx \sum_{\ell} (a_{n}^{(j+\ell)} - a^{(j+\ell)}) P(\varpi_n - \varsigma^{(j+\ell)} - \ell \varpi_\Delta) + (b_{n}^{(j)} - b^{(j)}) \\
- \sum_{\ell} (\hat{a}_{n}^{(j+\ell)} - a^{(j+\ell)}) P(\varpi_n - \varsigma^{(j+\ell)} - \ell \varpi_\Delta) - (\hat{b}_{n}^{(j)} - b^{(j)}) \\
+ \sum_{\ell} (\varsigma^{(j+\ell)} - \varsigma^{(j+\ell)}) a^{(j+\ell)} \hat{P}(\varpi_n - \varsigma^{(j+\ell)} - \ell \varpi_\Delta)
\]  

(93)

Demanding

\[
\frac{\partial J^{(j)}}{\partial(\varsigma^{(j)}, \hat{a}^{(j)}, \hat{b}^{(j)})} = 0
\]  

(94)

yield the following set of equations:

\[
0 = \sum_{n} W_n \left\{ \hat{P}(j) - \hat{P}(j) \right\} \hat{P}(\varpi_n - \varsigma^{(j)})
\]  

(95)

\[
0 = \sum_{n} W_n \left\{ \hat{P}(j) - \hat{P}(j) \right\} P(\varpi_n - \varsigma^{(j)})
\]  

(96)

\[
0 = \sum_{n} W_n \left\{ \hat{P}(j) - \hat{P}(j) \right\} .
\]  

(97)

By introducing the following short-hand notation:

\[
P^{(j)} = P(\varpi_n - \varsigma^{(j)} - \ell \varpi_\Delta).
\]  

(98)

and letting \( j = 0 \), we get:

\[
\sum_{\ell} \Delta \varsigma^{(\ell)} a^{(\ell)} \sum_{n} W_n \hat{P}^{(0)} - \sum_{\ell} \Delta \hat{a}^{(\ell)} \sum_{n} W_n \hat{P}^{(0)} P_n^{(\ell)} - \Delta \hat{b}^{(0)} \sum_{n} W_n \hat{P}^{(0)} \\
= - \sum_{n} W_n \hat{P}^{(0)} \left\{ \sum_{\ell} \Delta a_{n}^{(\ell)} P_n^{(\ell)} + \Delta b_{n}^{(0)} \right\}
\]  

(99)

\[
\sum_{\ell} \Delta \varsigma^{(\ell)} a^{(\ell)} \sum_{n} W_n P_n^{(\ell)} \hat{P}^{(0)} - \sum_{\ell} \Delta \hat{a}^{(\ell)} \sum_{n} W_n P_n^{(\ell)} P_{n}^{(0)} - \Delta \hat{b}^{(0)} \sum_{n} W_n P_{n}^{(0)} \\
= - \sum_{n} W_n P_{n}^{(0)} \left\{ \sum_{\ell} \Delta a_{n}^{(\ell)} P_{n}^{(\ell)} + \Delta b_{n}^{(0)} \right\}
\]  

(100)

\[
\sum_{\ell} \Delta \varsigma^{(\ell)} a^{(\ell)} \sum_{n} W_n \hat{P}^{(\ell)} - \sum_{\ell} \Delta \hat{a}^{(\ell)} \sum_{n} W_n P_{n}^{(\ell)} - \Delta \hat{b}^{(0)} \sum_{n} W_n \\
= - \sum_{n} W_n \left\{ \sum_{\ell} \Delta a_{n}^{(\ell)} P_{n}^{(\ell)} + \Delta b_{n}^{(0)} \right\}.
\]  

(101)
**Mean value:** If the mean value is taken on both sides of the equation set above, we get three equations for each \( j \). Since all the equations are zero on the right hand side, and we have the same number of unknowns as equations, we get

\[
E\{\hat{\varsigma}(j)\} = \varsigma(j), \quad E\{\hat{a}(j)\} = a(j), \quad E\{\hat{b}(j)\} = b(j).
\]

(102)

for all \( j \).

**Estimator variance:** If we drop the superscript when \( \ell = 0 \), and impose the following two constrains:

\[
\sum_n W_n \hat{P}_n P_n = 0, \quad \sum_n W_n \hat{P}_n = 0,
\]

(103)

the equation set can be rewritten as

\[
-\Delta \hat{\varsigma} a \sum_n W_n \hat{P}_n^2 = \sum_n W_n \hat{P}_n \{u_n + q_n\}
\]

(104)

\[
\Delta \hat{a} \sum_n W_n P_n^2 + \Delta \hat{b} \sum_n W_n P_n = \sum_n W_n \{u_n + q_n\}
\]

(105)

\[
\Delta \hat{a} \sum_n W_n P_n + \Delta \hat{b} \sum_n W_n = \sum_n W_n \{u_n + q_n\}
\]

(106)

where

\[
u_n = \sum_\ell \Delta a_\ell P_\ell, \quad q_n = \sum_{\ell \neq 0} \{\Delta \varsigma_\ell \hat{a}_\ell P_\ell - \Delta \hat{a}_\ell P_\ell\}.
\]

(107)

Solving the set of equations, yield

\[
\Delta \hat{\varsigma} = -\frac{\sum_n W_n \hat{P}_n \{u_n + q_n\}}{a \sum_n W_n \hat{P}_n^2}
\]

(108)

\[
\Delta \hat{a} = \frac{\sum_n W_n \{M_{22} P_n - M_{12}\} \{u_n + q_n\}}{\det\{M\}}
\]

(109)

\[
\Delta \hat{b} = \frac{\sum_n W_n \{M_{11} - M_{21} P_n\} \{u_n + q_n\}}{\det\{M\}}
\]

(110)

where \( M_{jk} \) are the elements of the following matrix:

\[
M = \left[ \begin{array}{cc}
\sum_n W_n P_n^2 & \sum_n W_n P_n \\
\sum_n W_n P_n & \sum_n W_n 
\end{array} \right].
\]

(111)

In the computation of the of the variance we will treat \( (\Delta \varsigma_\ell, \Delta \hat{a}_\ell, \Delta \hat{b}_\ell) \), for \( \ell \neq 0 \), deterministic (all in \( q \)). That means that we will under-estimate the variances. Later we
will make a conservative estimate for the errors made. The variance of \( \hat{\varsigma} \) way be written as
\[
\sigma^2_{\hat{\varsigma}} = \frac{1}{NMr},
\]
where \( N \) is the number of samples of the profiles, \( M \) is the number of averages done creating the profile. The estimator performance number:
\[
r = \frac{\left\{ \frac{1}{N} \sum_n W_n \hat{P}_n^2 \right\}^2}{\frac{1}{N} \sum_n W_n^2 \hat{p}_n^2 \chi_n^2},
\]
and the normalized noise variance:
\[
\chi_n^2 = \sum_{\ell} \mu_{\ell}^2 \{ P_{n}(\ell) \}^2 + \rho^2, \quad \rho = \frac{b}{a}, \quad \mu_{\ell} = \frac{a^{(\ell)}}{a},
\]
are numbers, independent of \( M \) and \( N \) (asymptotic). The variance of \( u_n \) is related to \( \chi_n^2 \) in the following way
\[
\sigma^2_{u_n} = \frac{a}{M} \chi_n^2.
\]

**Optimal weight function:** Finding the optimal weight function is the same as maximizing \( r \) with the constraints given by equation (103). By defining the following function:
\[
\tilde{r} \equiv r + \frac{2\lambda_1}{N} \sum_n W_n \hat{P}_n P_n + \frac{2\lambda_2}{N} \sum_n W_n \hat{P}_n.
\]
where the last terms contains products between two Lagrange multipliers \( (\lambda_1, \lambda_2) \) and the constrain conditions. Solving the minimizing problem is the same as solving:
\[
\frac{\partial \tilde{r}}{\partial W_m} = 0, \quad \frac{\partial \tilde{r}}{\partial (\lambda_1, \lambda_2)} = 0.
\]
First we rewrite equation (116) as
\[
\tilde{r} \frac{1}{N} \sum_n W_n^2 C_n = \left\{ \frac{1}{N} \sum_n W_n B_n^{(0)} \right\}^2 + \frac{2\lambda_1}{N} \sum_n W_n B_n^{(1)} + \frac{2\lambda_2}{N} \sum_n W_n B_n^{(2)}
\]
where
\[
C_m \equiv \hat{P}_m^2 \chi_m^2, \quad B_m^{(1)} \equiv \hat{P}_m P_m, \quad B_m^{(2)} \equiv \hat{P}_m.
\]
Then perform the operation \( \partial / \partial W_m \) on both sides, yielding the following expression for the weight function:
\[
W_m = \frac{1}{\tilde{r}} \sum_{k=0}^{2} \lambda_k \frac{B_m^{(k)}}{C_m},
\]
where the definition
\[ \lambda_0 = \frac{1}{N} \sum_n W_n B_n^{(0)} \] (122)
is used. If we now multiplies equation (121) with \( B_m^{(j)} \) \((j = 0, 1, 2)\), followed by a summation over \( m \) divided by \( N \) we get
\[ B^{(0,0)} + \tilde{\lambda}_1 B^{(1,0)} + \tilde{\lambda}_2 B^{(2,0)} = r \] (123)
\[ B^{(0,1)} + \tilde{\lambda}_1 B^{(1,1)} + \tilde{\lambda}_2 B^{(2,1)} = 0 \] (124)
\[ B^{(0,2)} + \tilde{\lambda}_1 B^{(1,2)} + \tilde{\lambda}_2 B^{(2,2)} = 0 \] (125)
where \( \tilde{\lambda}_k = \lambda_k / \lambda_0 \) and
\[ B^{(j,\ell)} = \frac{1}{N} \sum_m \frac{B_m^{(j)} B_m^{(\ell)}}{C_m} \] (126)
From the two last lines (eq. (124) and (125)) \((\tilde{\lambda}_1, \tilde{\lambda}_2)\) can be found, and from the first line we get
\[ r = \frac{1}{N} \sum_n \frac{\dot{P}_n^2 + \tilde{\lambda}_1 \dot{P}_n P_n + \tilde{\lambda}_2 \dot{P}_n}{\chi_n^2} \] (127)

**Special case:** One special case is when \( \chi_m^2 \) is symmetrical, then both \( B^{(0,1)} \) and \( B^{(0,2)} \) is zero, and from equation (124) and (125) we have the solution: \( \tilde{\lambda}_1 = \tilde{\lambda}_2 = 0 \). This special case occurs when the intensity in the side-bands are symmetrical: \( \mu_\ell = \mu_{-\ell} \), yielding
\[ W_m \propto \frac{1}{\chi_m^2} \frac{\dot{P}_n^2}{r} = \frac{1}{N} \sum_n \frac{\dot{P}_n^2}{\chi_n^2} \] (128)
This is the optimal solution of the problem and the value of \( r \) represents the maximum possible performance number.
C Regular gridded 2D-antenna

Ideal form: Let $L^a$ and $L^e$ be the distance between the $(N^a \times N^e)$ elements in azimuth and elevation direction, respectively, and let the size of the elements be equal to the distance between the elements. If the elements are radiation uniformly the pattern from one element is given by

$$
\int \int d\ell \frac{e^{ikr\sqrt{x^2 + y^2}}}{4\pi i \sqrt{x^2 + y^2}} \approx \frac{e^{ikr}}{4\pi ir} \int \int d\ell e^{-ikr} \frac{L^a L^e}{4} \sin \frac{\zeta^a}{2} \sin \frac{\zeta^e}{2},
$$

(129)

where

$$
r = \sqrt{x^2 + \|y\|^2}
$$

(130)
is the distance between target and antenna center, and

$$
\zeta^a \equiv \frac{kL^a}{r} y^a = k_r L^a \sin \Phi, \quad \zeta^e \equiv \frac{kL^e}{r} y^e = k_r L^e \sin \Theta.
$$

(131)

Here is $\Phi$ the azimuth angle and $\Theta$ the elevation-angle. The full two-way antenna pattern can be written as

$$
A(\zeta; \zeta_0) = A^t(\zeta; \zeta_0) A^r(\zeta; \zeta_0)
$$

(132)

where the transmision and receive pattern are given by

$$
A^{t/r}(\zeta; \zeta_0) = f(\zeta) \sum_{m,n} Z^{t/r}_{m,n}(\zeta_0) e^{i(\zeta - \zeta_0)(m,n)}.
$$

(133)

and $\zeta = (\zeta^a, \zeta^e)$. The antenna steering direction is given by $\zeta_0$ and

$$
f(\zeta) \equiv \sin \frac{\zeta^a}{2} \sin \frac{\zeta^e}{2},
$$

(134)

represents the 2D sub-antenna pattern. Since $f$ is dyadic in form (azimuth and elevation direction are separable), the full antenna can also be dyadic if the reflection and transmission coefficients are chosen dyadic: $Z^{t/r}_{m,n}(\zeta) = z^{at/r}_{m}(\zeta^a) z^{ar/r}_{n}(\zeta^e)$.

General form: In the real world, the sub-antenna pattern are not necessarily dyadic and the individual patterns are not necessarily equal, neither as function of position, polarization or transmission/receive direction. The transmission and receive pattern for the full antenna, then takes the following general form:

$$
A^{t/r}(\zeta; \zeta_0) = \sum_{m,n} Z^{t/r}_{m,n}(\zeta_0) e^{i(\zeta - \zeta_0)(m,n)} f^{t/r}_{m,n}(\zeta).
$$

(135)
C.1 1D beam forming

In this section it is assumed that the transmitted antenna pattern is given, and optimization is only done for the receive pattern. Minimizing the following cost-function:

\[ J = \int_{\mathbb{R}\setminus L(\zeta_0)} d\zeta \left| f(\zeta) \sum_{n} z_n^r e^{i(\zeta - \zeta_0)n} \right|^2 \left| A^t(\zeta; \zeta_0) \right|^2 + \lambda \left\{ \int_{L(\zeta_0)} d\zeta \left| f(\zeta) \sum_{n} z_n^r e^{i(\zeta - \zeta_0)n} \right|^2 \left| A^r(\zeta; \zeta_0) \right|^2 - 1 \right\} , \]  

(136)

where \( L(\zeta_0) \) is the interval \((\zeta_0 - \zeta_\Delta/2, \zeta_0 + \zeta_\Delta/2)\), yield

\[ \sum_{n} \{ \hat{p}_{n-m} a_n^r - \hat{q}_{n-m} b_n^r \} + \lambda \sum_{n} \{ \hat{p}_{n-m} a_n^r - \hat{q}_{n-m} b_n^r \} = 0 \]  

(137)

\[ \sum_{n} \{ \hat{q}_{n-m} a_n^r + \hat{p}_{n-m} b_n^r \} + \lambda \sum_{n} \{ \hat{q}_{n-m} a_n^r + \hat{p}_{n-m} b_n^r \} = 0 \]  

(138)

where \( a_n^r, b_n^r \) are the real and imaginary part of the transmission coefficients:

\[ z_n^r \equiv a_n^r + i b_n^r , \]  

(139)

and

\[ \hat{p}_{n-m} = \int_{L(\zeta_0)} d\zeta \left| A^t(\zeta; \zeta_0) f(\zeta) \right|^2 \cos((\zeta - \zeta_0)(n-m)) , \]

\[ \hat{q}_{n-m} = \int_{L(\zeta_0)} d\zeta \left| A^t(\zeta; \zeta_0) f(\zeta) \right|^2 \sin((\zeta - \zeta_0)(n-m)) , \]  

(139)

\[ \hat{p}_{n-m} = \int_{\mathbb{R}\setminus L(\zeta_0)} d\zeta \left| A^t(\zeta; \zeta_0) f(\zeta) \right|^2 \cos((\zeta - \zeta_0)(n-m)) , \]

\[ \hat{q}_{n-m} = \int_{\mathbb{R}\setminus L(\zeta_0)} d\zeta \left| A^t(\zeta; \zeta_0) f(\zeta) \right|^2 \sin((\zeta - \zeta_0)(n-m)) . \]

By defining the following matrix

\[ \hat{P} = \begin{bmatrix} \hat{p}_0 & \hat{p}_1 & \hat{p}_2 & \cdots & \hat{p}_{N-1} \\ \hat{p}_{-1} & \hat{p}_0 & \hat{p}_1 & \cdots & \hat{p}_{N-2} \\ \hat{p}_{-2} & \hat{p}_{-1} & \hat{p}_0 & \cdots & \hat{p}_{N-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{p}_{-N} & \hat{p}_{-N+1} & \hat{p}_{-N+2} & \cdots & \hat{p}_0 \end{bmatrix} \]  

(141)

and using the same convention for defining \( \hat{P}, \hat{Q} \) and \( \hat{Q} \), from the coefficients \( \hat{p}_{n-m}, \hat{q}_{n-m} \) and \( \hat{q}_{n-m} \), respectively, we may write the system as

\[ \{ \hat{A} - \lambda \hat{A} \} x^T = 0 , \]  

(142)

where

\[ \hat{A} \equiv \begin{bmatrix} \hat{P} & -\hat{Q} \\ \hat{Q} & \hat{P} \end{bmatrix} , \quad \hat{A} \equiv \begin{bmatrix} \hat{P} & -\hat{Q} \\ \hat{Q} & \hat{P} \end{bmatrix} , \quad x \equiv [a_0^r, \ldots, a_{N-1}^r, b_0^r, \ldots, b_{N-1}^r] . \]  

(143)
C.2 Doppler offsets caused by error in the antenna form

The azimuth direction 1D-problem is first considered. The Fourier-profile can be written as:

\[ P(\zeta - \zeta; x) = |A(\zeta - \zeta; \zeta_0)|^2. \]  

(144)

Here is \( x \) given as the collection of the real and imaginary part of the transmission and receive coefficients:

\[ x = (a_0, \ldots, a_{N\Delta - 1}, b_0, \ldots, b_{N\Delta - 1}, a_0^\ast, \ldots, a_{N\Delta - 1}^\ast, b_0^\ast, \ldots, b_{N\Delta - 1}^\ast) \]  

(145)

The goal is to find an estimate \( \zeta \) of the true offset \( \zeta \), given only an estimate \( \hat{x} \) of the true vector \( x \). This yields the following minimizing problem:

\[ 0 = \frac{\partial}{\partial \zeta_0} \int d\zeta W(\zeta) \left\{ P(\zeta - \zeta; x) - P(\zeta - \zeta; \hat{x}) \right\}^2 \]  

(146)

where in the computations, without loss of generality, is assumed \( \zeta \) to be zero. By writing

\[ P(\zeta - \zeta; \hat{x}) \approx P(\zeta; x) + (\hat{x} - x) \cdot \nabla P(\zeta; x) - \zeta \hat{P}(\zeta; x), \]  

(147)

the minimization problem becomes

\[ 0 = \int d\zeta W(\zeta) \left\{ (\hat{x} - x) \cdot \nabla P(\zeta; x) - \zeta \hat{P}(\zeta; x) \right\} \hat{P}(\zeta; x) \]  

(148)

Solved with respect to \( \zeta \), we get

\[ \zeta = (\hat{x} - x) \cdot \frac{\int d\zeta W(\zeta) \nabla P(\zeta; x) \hat{P}(\zeta; x)}{\int d\zeta W(\zeta) \hat{P}(\zeta; x)} \]  

(149)

This gives the following expression for the variance of \( \zeta \) as function of the variance of the estimated transmission and reflection coefficients:

\[ \sigma_{\zeta}^2 = \sum_{\ell} \sigma_{\zeta_{\ell}}^2 \left\{ \frac{\int d\zeta W(\zeta) \nabla_{\zeta} \hat{P}(\zeta; x)}{\int d\zeta W(\zeta) \hat{P}(\zeta; x)} \right\}^2. \]  

(150)

The response of an error in the coefficients on the profile is given as

\[ \frac{\partial P}{\partial a_{n/\tau}^l} = \frac{B_{n/\tau}^l(\zeta; \zeta_0) + B_{l/\tau}^l(\zeta; \zeta_0)}{2} = \Re\{B_{n/\tau}^l(\zeta; \zeta_0)\}, \]  

(151)

\[ \frac{\partial P}{\partial b_{n/\tau}^l} = \frac{B_{n/\tau}^l(\zeta; \zeta_0) - B_{l/\tau}^l(\zeta; \zeta_0)}{2i} = \Im\{B_{n/\tau}^l(\zeta; \zeta_0)\}, \]  

(152)

where

\[ B_{n/\tau}^l(\zeta; \zeta_0) \equiv 2A^l(\zeta; \zeta_0) f^{l/\tau}(\zeta) e^{-i(\zeta - \zeta_0)n} |A^{l/\tau}(\zeta; \zeta_0)|^2. \]  

(153)
By the following definition

\[ p_{n}^{t/r} = \frac{\int d\zeta W(\zeta) B_{n}^{t/r}(\zeta; \zeta_{0}) \hat{\dot{P}}(\zeta; x)}{\int d\zeta W(\zeta) P^{2}(\zeta; x)} \]  \hspace{1cm} (154)

can \( \zeta \) be written as

\[ \zeta = \sum_{n} \left\{ (\hat{a}_{n}^{i} - a_{n}^{i}) \Re\{p_{n}^{i}\} + (\hat{b}_{n}^{i} - b_{n}^{i}) \Im\{p_{n}^{i}\} + (\hat{a}_{n}^{r} - a_{n}^{r}) \Re\{p_{n}^{r}\} + (\hat{b}_{n}^{r} - b_{n}^{r}) \Im\{p_{n}^{r}\} \right\} \]  \hspace{1cm} (155)

or even more compact as

\[ \zeta = \sum_{n} \Re\{ (\hat{z}_{n}^{i} - z_{n}^{i}) p_{n}^{i\ast} + (\hat{z}_{n}^{r} - z_{n}^{r}) p_{n}^{r\ast} \} , \]  \hspace{1cm} (156)

and if the variance of the real and imaginary part of each t/r-coefficient are equal, the expression for the variance simplifies to

\[ \sigma_{\zeta}^{2} = \sum_{n} \left\{ \sigma_{\hat{a}_{n}}^{2} |p_{n}^{i}|^2 + \sigma_{\hat{a}_{n}}^{2} |p_{n}^{r}|^2 \right\} . \]  \hspace{1cm} (157)
D Figures

This section shows figures related to Appendix B on the expected performances.

In Figure 9 is shown the effect of aliasing on the baseband azimuth frequency profile for different fraction (κ) of the bandwidth processed.

In Figure 10 is shown the estimated Doppler bias as function of fraction of bandwidth for different SNR, where for the last three plots weight functions scaled to the SNR are used.

In Figure 11 is the estimator performance (eq. (127)) versus SNR plotted for different fraction of bandwidth and weight functions.

In Figure 12 is the Doppler offset standard deviation (Hz) (eq. (112 and eq. (127)) estimated over a given number of pixels shown versus SNR for different fractions of bandwidth and weight functions.

In Figure 13 is shown the Doppler width standard deviation (Hz) (eq. (115)) estimated over a given number of pixels shown versus Doppler width for different SNR.

The figures show that the requirement for the Doppler offset accuracy (5Hz) can be achieved.
Figure 9: Azimuth Fourier profiles, where the aliased profile is in black, the zero-band profile in red and the additive noise level in green. The different plots are for $\kappa = (0.6, 0.7, 0.8, 0.9)$, all with a 3 dB increase in energy of first side-band.
Figure 10: Doppler bias as function of $\kappa$, caused by a 3 dB non-corrected increase in first side-band (solid line), second side-band (dashed line) and third side-band (dotted line). Plot (a) represents the flat weight function and plot (b), (c) and (d) are for different SNR with the $W_n = 1/\chi_n^2$ weight-function (PRF = 1650 Hz in all plots).
Figure 11: Estimator performance as function of additive noise level. The different lines represents $\sqrt{r}$ for different values of $\kappa$ (the upper lines represents $\kappa = 0.9$). The solid lines represents the performance of using the $W_n = 1/\chi_n^2$ weight function, and the dashed lines, the performance of using constant weights.

Figure 12: Doppler standard deviation as function of additive noise. The different lines represents different values of $\kappa$ (the lower lines represents $\kappa = 0.9$). The solid lines represents the use of the $W_n = 1/\chi_n^2$ weight function, and the dashed lines, the use of constant weights.
Figure 13: Doppler width standard deviation as function of Doppler width. Upper line represents SNR=0 dB and lower line SNR=24 dB.